

U-Statistics for Multiple Censored Outcomes with Varying Frequency, Severity, Attribution



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Doctors Fear Losing Leukemia Drug Deemed Risky

GRADY D, POLLAK A (2013) *NYTimes* Nov 1:A3

An FDA spokeswoman said that studies of the drug to date involved 530 patients of which

2.6 percent (14) had

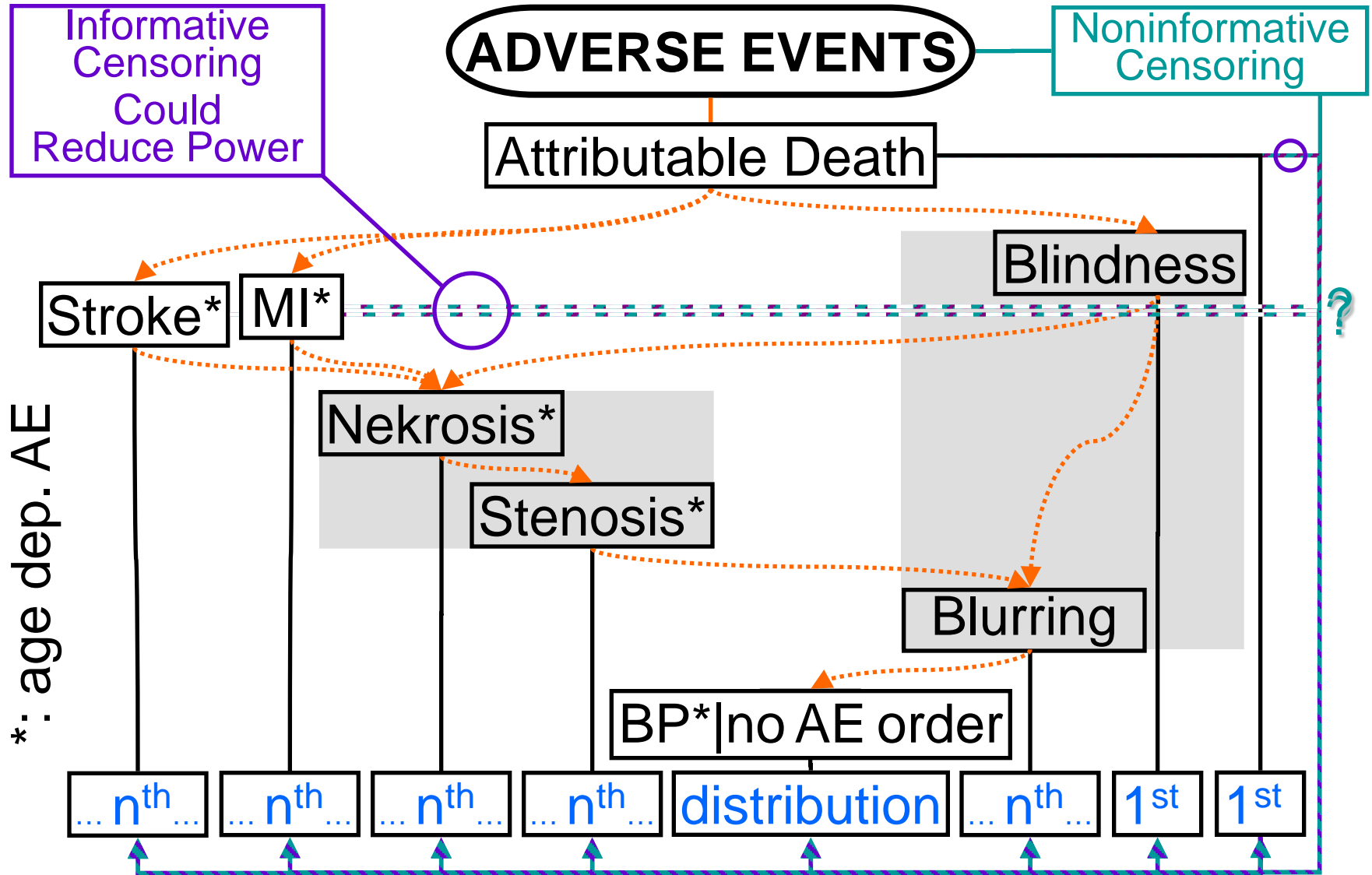
- *died from cardiovascular problems.*

24 percent taking Iclusig for a median of 1.3 years, and 48 percent studied for a median of 2.7 years, had suffered serious adverse vascular events, starting at 2 weeks of treatment, including

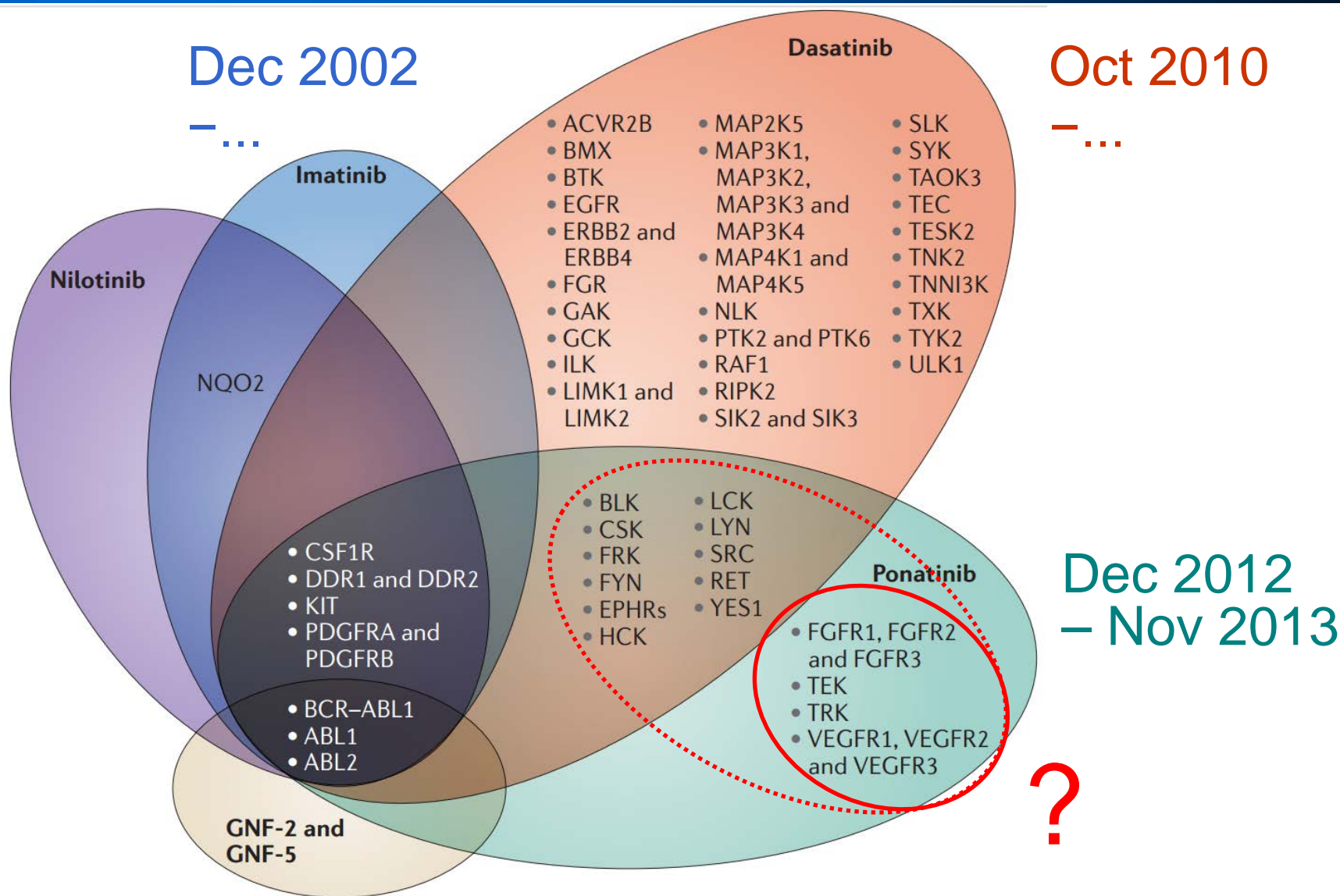
- *heart attacks,*
- *strokes,*
- *[blurred vision /] blindness, and*
- *lack of blood flow to [/ tissue death in] the extremities;*
- *high blood pressure occurred in 67 percent]*



Complex Outcomes



Targets of BCR-ABL Inhibitors



Abstract

When several (repeated, censored) events graded by severity and/or attribution need to be considered (e.g., hospitalization, myocardial infarction (MI), stroke, death), many strategies currently discussed either are not comprehensive enough or require unrealistic assumptions to be made, with often unpredictable consequences for validity.

We present a flexible approach to generalise the Gehan / Wei / Knuiman test to incorporate information about complex ordinal and hierarchical/factorial relationships between outcomes.

The proposed approach fulfills the ICH E9 criteria for “a predefined algorithm”, yet is flexible enough to objectively incorporate information about graded, repeated, and censored events for a wide range of complex diseases.

Overview

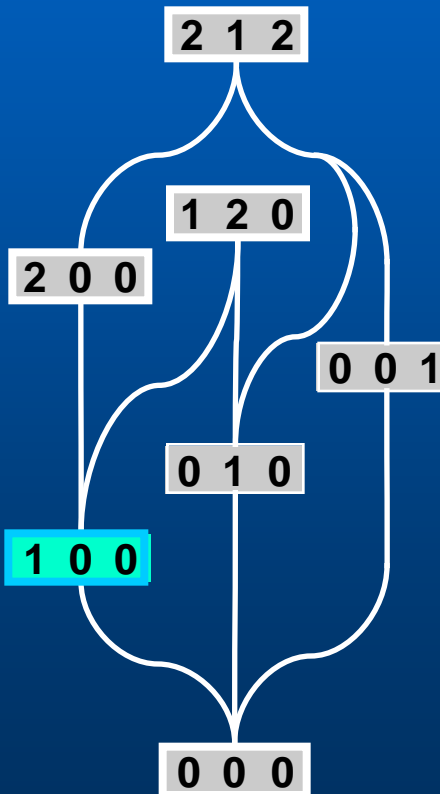
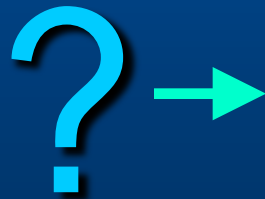
- U-statistics for multivariate data
 - Theory
 - Tools
 - Examples
- Increasing Information Content (IC)
 - AE / Response Structures
 - Censoring
 - Relatedness / Attribution
 - Grading by Severity
 - Genetic Structures
- “Gold mining” Phase II Trial Data

Multivariate Outcomes

Death
MI
Stroke

Partial Ordering
Lattice

sCD25
IL1b
CD68/CD2



Efficacy
Cost
Safety

SNP A / B / C

Environment
Society
Patient

300 Years of Statistics: From Notepads to Web Tools

John Arbuthnot (1667–1735)

1667: * Scotland
1692: translated HUYGENS' (1657)
De ratiociniis in ludo aleae
(1st English work on prob')
1696: MD from St Andrews Univ.,
taught **mathematics**
1704: Fellow of the *Royal Society*
1705: **physician** to Queen Anne
1714: founded the *Scriblerus Club*
to satirise bad poetry and
pedantry with
T. PARNELL, J. GAY,
A. POPE, and J. SWIFT
(1729: *A modest proposal*).
1735: † London

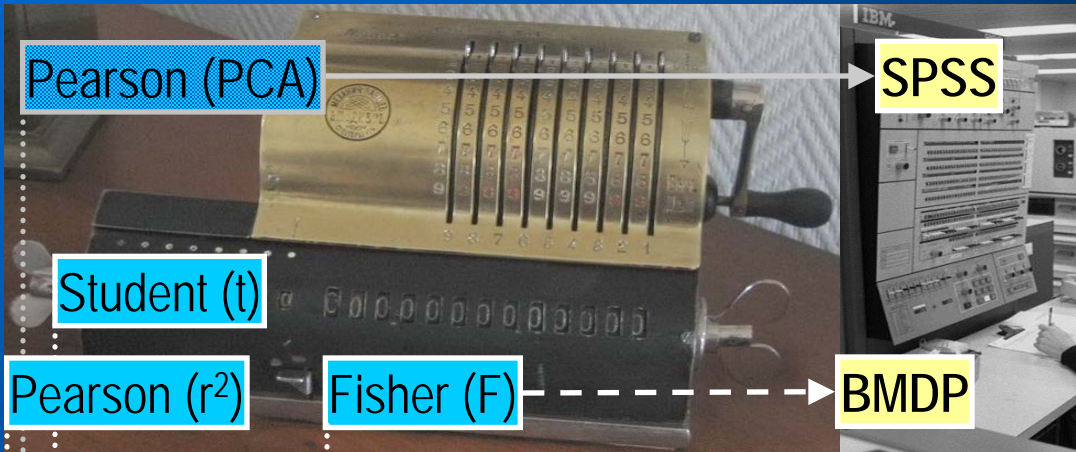
(1692) *On the laws of chance*

- discussing the slight excess of male births over female births
- demonstrating that **not chance** causes the sex ratio at birth
- the first application of probability to social statistics / medicine.

For „realistic“ data (rounded, inexact, etc.), versions of this „**sign test**“ were developed 250 years later ...

A Short History of Statistics

Linear Model



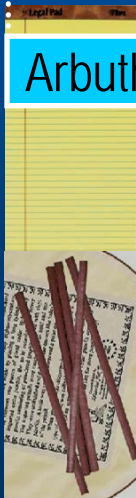
One of the attributes of ANOVA which ensured its **early popularity** was **computational elegance**. The structure of the additive model leads to **simple algebra** rather than **matrix calculations**.

en.wikipedia.org/wiki/Analysis_of_variance (2013-02-23)

W3 WXP/OSX
16 MB 3 GB



U-Statistics



Arbuthnot (sign test)

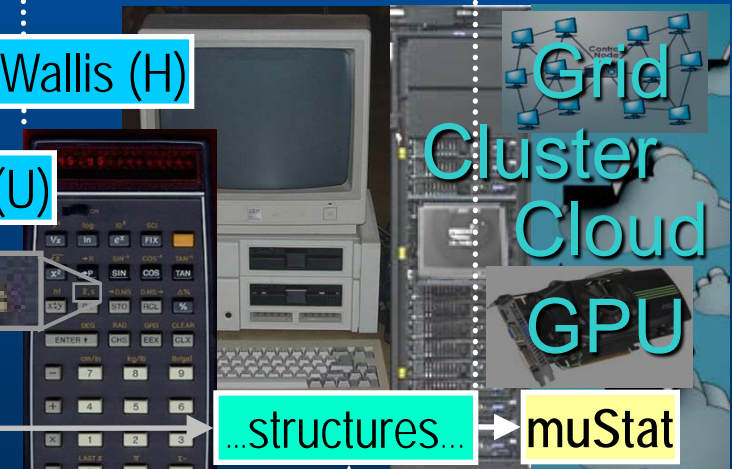
One of the attributes of U-statistics, which ensures **biological validity** of results is its **conceptual simplicity**. The **matrix calculations** required, however, are **computationally demanding**.

Kendall (τ)

Kruskal-Wallis (H)

Mann-Whitney (U)

Hoeffding



...structures...

Computational Biostatistics

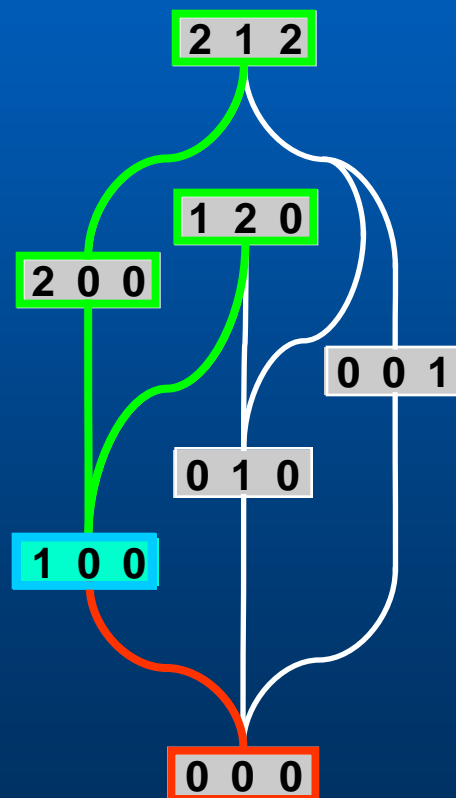
U-Statistics for Multivariate Data: The Basics

A Class of Statistics

u scores are computed from
 #(lower profiles)
 #(higher profiles).

$$u(100) = 1 - 3 = -2$$

Partial Ordering
Lattice



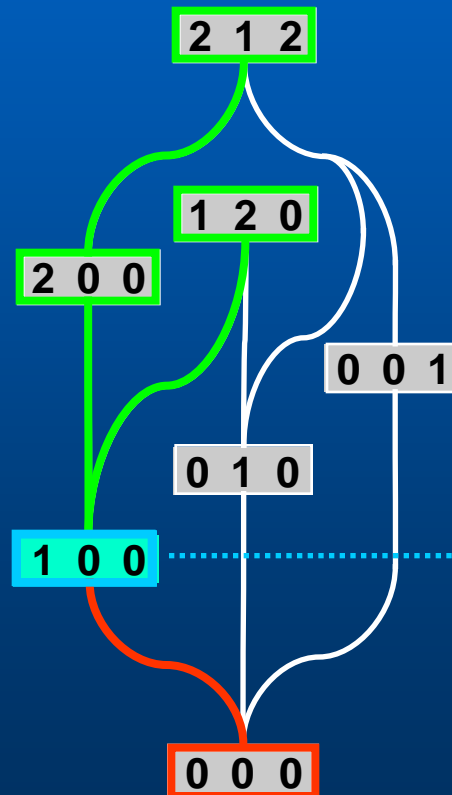
HOEFFDING W (1948) AMS 19:293

Algorithm (Deuchler, 1914):

u scores are computed as
 $\#(\text{lower profiles}) - \#(\text{higher profiles})$.

$$u(100) = 1 - 3 = -2$$

Partial Ordering Lattice



Pairwise Orderings

Combining several ordinal measures in clinical studies

Knut M. Wittkowski^{1,*}, Edmund Lee², Rachel Nussbaum²,
 Francesca N. Chamian² and James G. Krueger²

¹General Clinical Research Center, The Rockefeller University, New York, NY, U.S.A.
²Laboratory of Investigative Dermatology, The Rockefeller University, New York, NY, U.S.A.

		2	1	2	0	0	1	0	
		1	2	0	0	1	0	0	
		2	0	0	1	0	0	0	
	2	1	2	0	0	1	1	1	1
	1	2	0	0	0	0	1	1	1
	2	0	0	0	0	0	1	1	1
	0	0	1	0	0	0	0	0	1
	0	1	0	1	0	0	0	0	1
	1	0	0	-1	-1	-1	0	0	1
	0	0	0	-1	-1	-1	-1	-1	0
									5
									3
									1
									0
									-1
									-2
									-6

WITTKOWSKI KM (2004) *Stat Med* 23

K-variate Censored Data

Let X_{skj} denote the time to event k for subject j in group s .

We observe $(\bar{X}_{skj}, \delta_{skj})$, where $\bar{X}_{skj} = \begin{cases} (-\infty, \delta_{skj} = 1) \\ (U_{skj} (< X_{skj}), \delta_{skj} = 1) \\ (X_{skj}, \delta_{skj} = 0) \end{cases} \Leftrightarrow (X_{skj}^-, X_{skj}^+) = \begin{cases} (-\infty, \infty) \\ (X_{skj}, \infty) \\ [X_{skj}, X_{skj}] \end{cases}$.

WEI, KNUIMAN (1987) define a score statistic

$$I_{WK}(\mathbf{X}_{1i} \leq \mathbf{X}_{2j}) = \phi(\bar{X}_{1i}, d_{1i}; \bar{X}_{2j}, d_{2j}) = \begin{cases} 1 & \forall_k \bar{X}_{1ki} \leq \bar{X}_{2kj}, \delta_{1ki} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Censored Obs Only

“This type of order ... is rather strong. ... Several comparisons make no contribution.” [p. 218]

GEHAN (1965), MANTEL (1966), COX (1972), and WEI, LACHIN (1984) (in the notation of WITKOWSKI (2004)) define a less “strong” score statistic

$$I(\mathbf{X}_{1i} \leq \mathbf{X}_{2j}) = \phi(X_{1i}^-, X_{1i}^+, X_{2j}^-, X_{2j}^+) = \begin{cases} 1 & \forall_k X_{1ki}^+ \leq X_{2kj}^- \supset \delta_{1ki} = 0 \\ 0 & \text{otherwise} \end{cases}$$

No Overlap

The U statistic then is

$$u(\mathbf{X}_{1i}, \mathbf{X}_{2j}) = I(\mathbf{X}_{1i} \leq \mathbf{X}_{2j}) - I(\mathbf{X}_{1i} \geq \mathbf{X}_{2j}) \in \{-1, 0, 1, NA\}$$

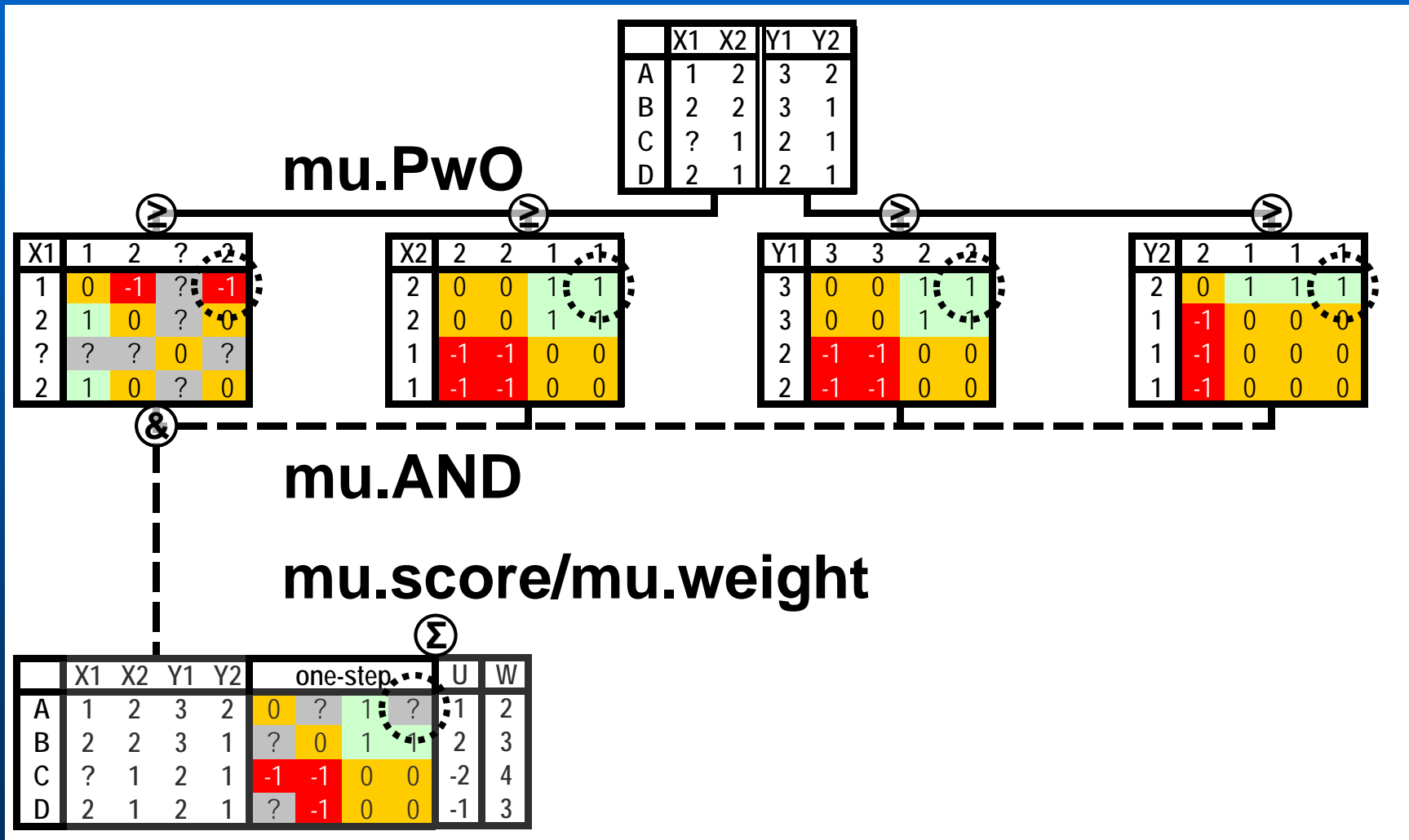
$$U_n = \frac{1}{n_1 n_2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} u(\mathbf{X}_{1i}, \mathbf{X}_{2j}) \stackrel{\text{as.}}{\sim} N(0, \hat{\sigma}_0) \quad s > 2, \text{ stratification: } W = \mathbf{u}'_+ \boldsymbol{\Sigma}_+^{-1} \mathbf{u}_+ \stackrel{\text{as.}}{\sim} \chi_{s-1}^2(0)$$

WITKOWSKI 1988

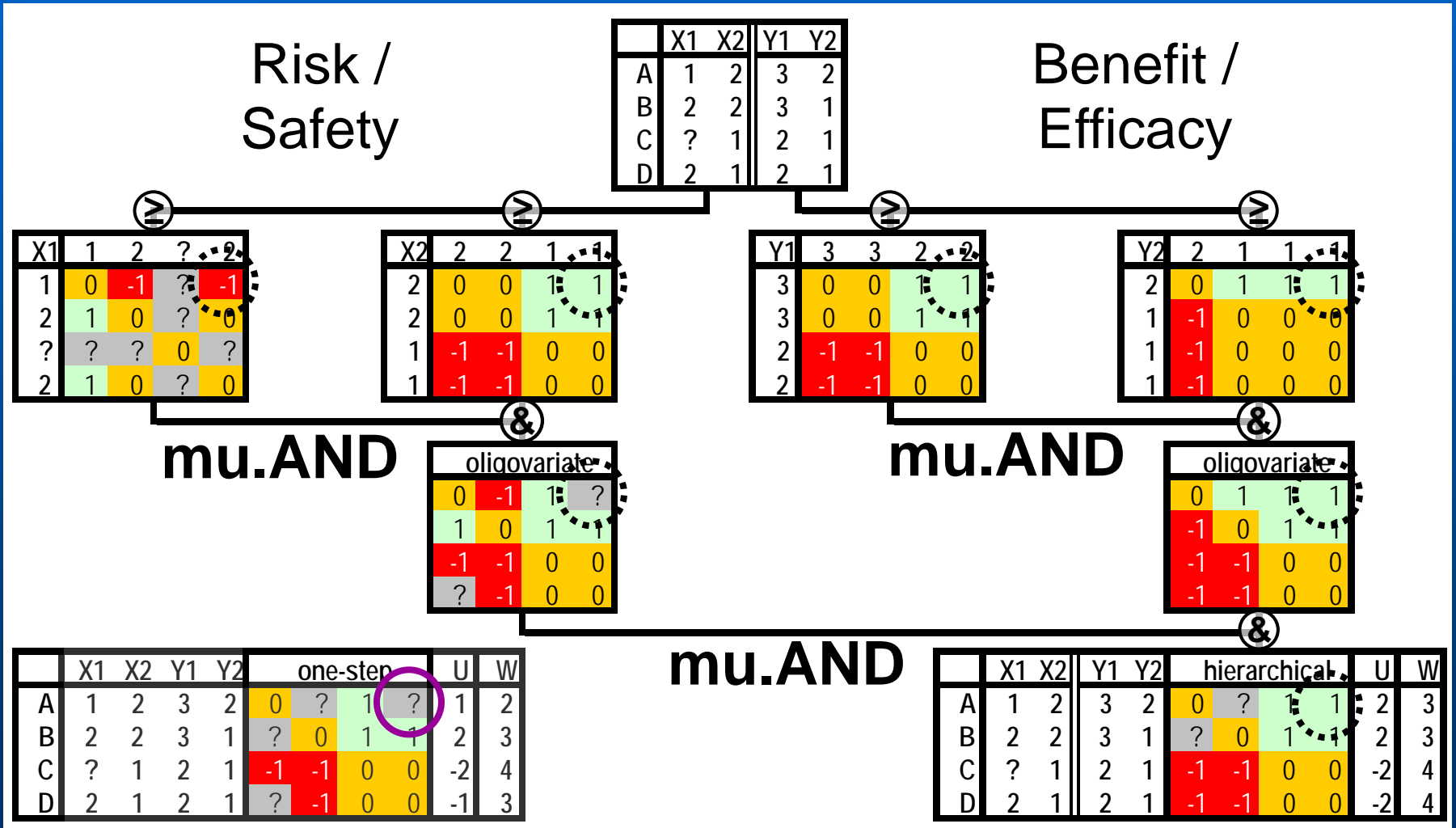
“The likelihood of indefiniteness ... will increase as the dimensionality of the data increases.”

U-Statistics for Multivariate Data: Structured Variables

#Variables vs Information content



Hierarchy Increases Information



Multiple Interval Censored Data

Time to MI

	X1	X2	Y1	Y2
A	1	2	2	3
B	2	2	1	3
C	?	1	1	2
D	1	2	1	2

Time to Stroke

X1	X2	oligovariate			U	
1	2	?	-1	1	?	0
2	2	1	0	1	1	3
?	1	-1	-1	?	-1	-3
1	2	?	-1	1	?	0

Y1	Y2	oligovariate			U	
2	3	?	?	1	1	2
1	3	?	?	?	?	0
1	2	-1	?	?	?	-1
1	2	-1	?	?	?	-1

	X1	X2	Y1	Y2	one-step		U	W		
A	1	2	2	3	0	?	1	1	2	3
B	2	2	1	3	?	0	1	1	2	3
C	?	1	1	2	-1	-1	0	-1	-3	4
D	1	2	1	2	-1	-1	1	0	-1	4

	X1	X2	Y1	Y2	hierarchical			U	W	
A	1	2	2	3	?	-1	1	1	1	3
B	2	2	1	3	1	0	1	1	3	4
C	?	1	1	2	-1	-1	?	-1	-3	3
D	1	2	1	2	-1	-1	1	?	-1	3

U-Statistics for Multivariate Data: Tools

S/R Functions

- **mu.GE**

pairwise comparison

- **mu.AND**

combine pairwise comparisons
allowing for hierarchical formula

- **mu.Sums**

generate scores and weights from
(combination of) pairwise comparisons

```

mu.GE <- function(x, y=x) { ...
  if (length(y)>1) apply(rbind(x, y), 2, mu.GE, nrow(x))
  else as.numeric(NAtoZer(outer(x[1:y], x[-(1:y)], ">="))) }

```

```

mu.AND <- function(GE, frml=NULL) { ...
  if (is.null(frml)) {
    GE <- sq.array(GE), AND <- GE[, , 1]^0, nNA <- AND[, 1]*0
    for (i in 1:dim(GE)[3]) {
      nNA <- nNA + diag(GEi <- GE[, , i])
      AND <- AND * (GEi + (1-GEi)*(1-t(GEi))) }
    return(as.numeric(AND * ((c(nNA)%c(nNA))>0))) }
  else # ... deal with the formula ...
  return(tmp[-1, 1]) }

```

```

mu.Sums <- function(GE, dsgn=1) {
  ICW <- function(GE, dsgn) {
    wgt <- colSums(GE|t(GE))
    sqrt(wgt*(wgt>1)/if (dsgn!=1) colSums(dsgn) else nrow(GE)}
  GE <- sq.matrix(GE)
  wght <- ICW(GE, dsgn)
  list (
    score = (rowSums(GE) - colSums(GE)) *ifelse(wght==0, NA, 1),
    weight = wght) }

```

mu.test

The new function
'**mu.test**' integrates /
extends
(at twice the speed)
several well-known
tests:

	Conditions	Granularity	Replications	Blocks
mcnemar.test	2	2	>1	1
SMN.test	2	2	>1	3
CA.test (new)	2	3	>1	1
wilcox.test	2	>1	>1	1
kruskal.test	>1	>1	>1	1
friedman.test	>1	>1	1	>1
mu.test	>1	>1	≥0	>1



CRAN

<http://cran.r-project.org/index.html>



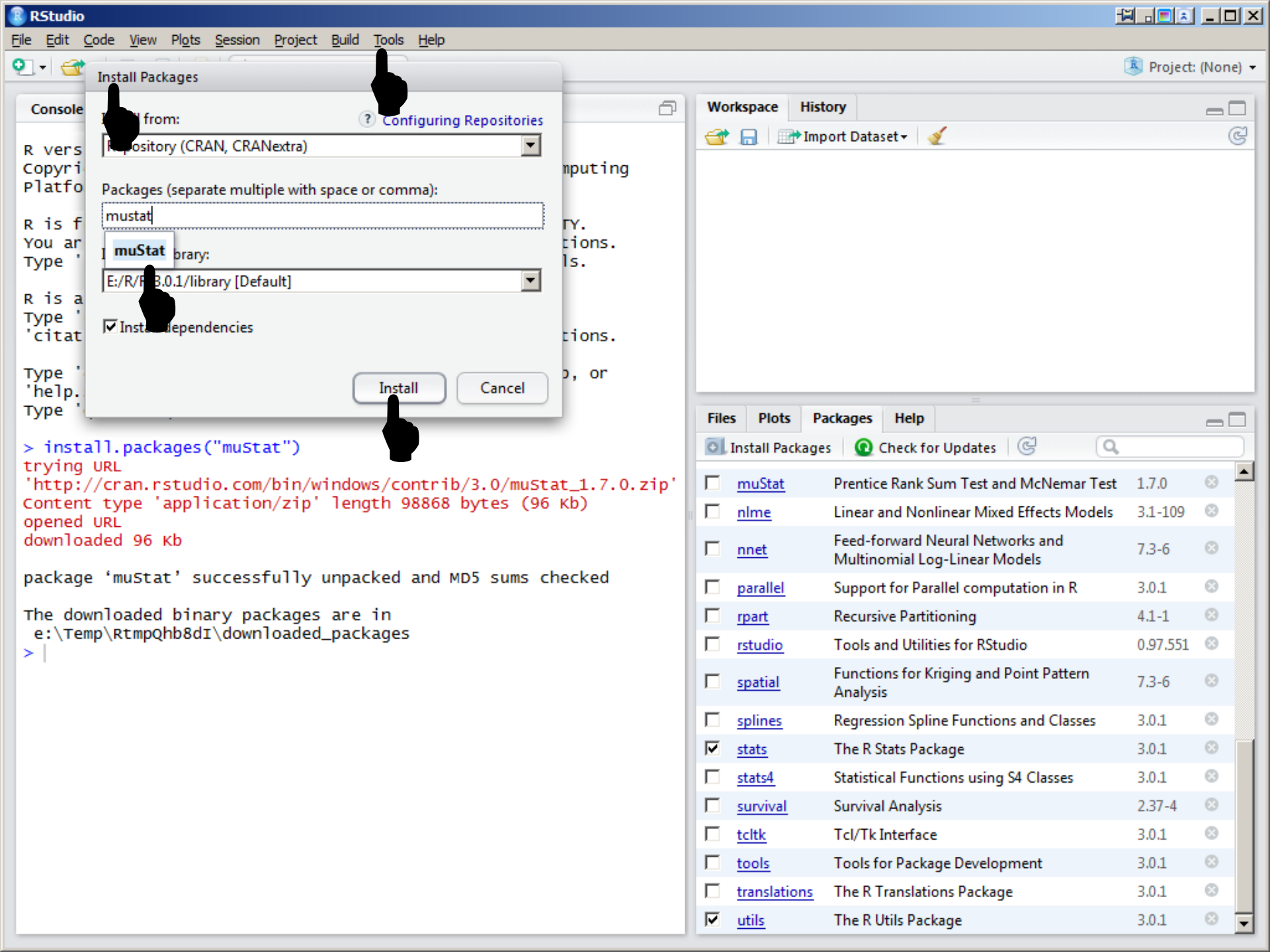
CSAN

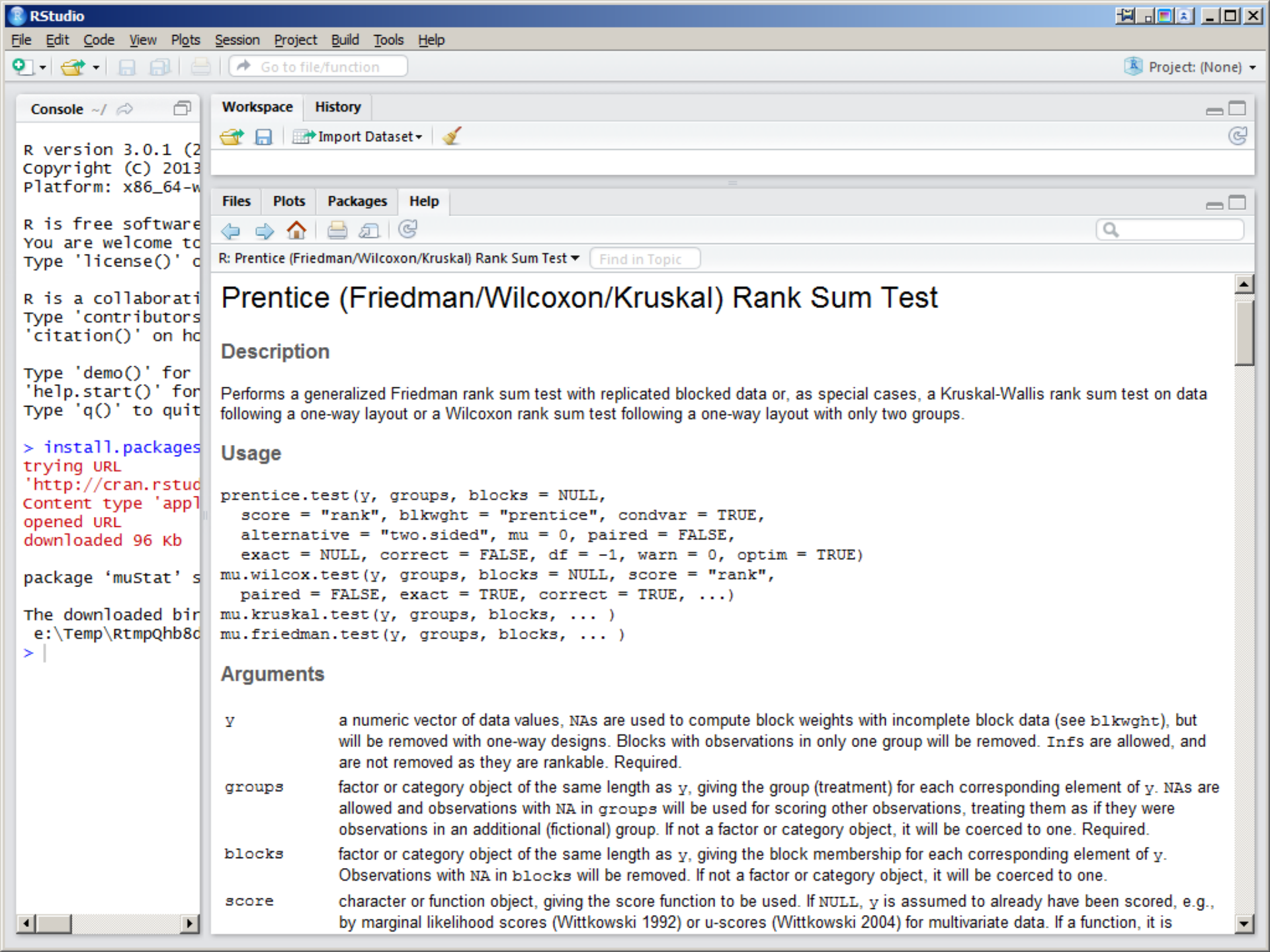
<http://csan.insightful.com/Default.aspx>

mu.test()



```
mu.test <- function(  
  y, # data (NA ok)  
  groups, # groups (unbalanced ok)  
  blocks = NULL, # blocks (unequal size ok)  
  score = "rank", # NULL: already scored  
  blkwght = "prentice", # block weights  
  # Wittkowski (1988) JASA  
  
  alternative = "two.sided", # wilcox only  
  mu = 0, # wilcox only  
  paired = FALSE, # wilcox only  
  exact = NULL, # wilcox only  
  correct = NULL # wilcox only  
)
```





```
R version 3.0.1 (2013-10-31)
Copyright (c) 2013 R Core Team
Platform: x86_64-w64-mingw32/x64
```

```
R is free software; you are free to copy,
distribute and modify it. You are
welcome to distribute it under the terms
of the GNU General Public License
Type 'license()' or 'licence()' for
more details.
R is a collaborative project with many
contributors. You are welcome to
contribute. Type 'contributors()' for
more details.
Type 'citation()' on how to cite R or
R packages in publications.
```

```
Type 'demo()' for some introductory
material, and 'help.start()' for an
HTML browser interface.
Type 'q()' to quit R.
```

```
> install.packages("muStat")
trying URL 'http://cran.rstudio.com/web/packages/muStat/muStat.pdf'
Content type 'application/pdf'
opened URL
downloaded 96 kb
```

```
package 'muStat' successfully
unpacked and installed to
'~/R/x86_64-w64-mingw32/x64/library/000/muStat'
The downloaded binary packages are in
~/Temp/RtmpQhb88c/library/000
```

```
>
```

Workspace History

Import Dataset

Files Plots Packages Help

R: Prentice (Friedman/Wilcoxon/Kruskal) Rank Sum Test Find in Topic

Prentice (Friedman/Wilcoxon/Kruskal) Rank Sum Test

Description

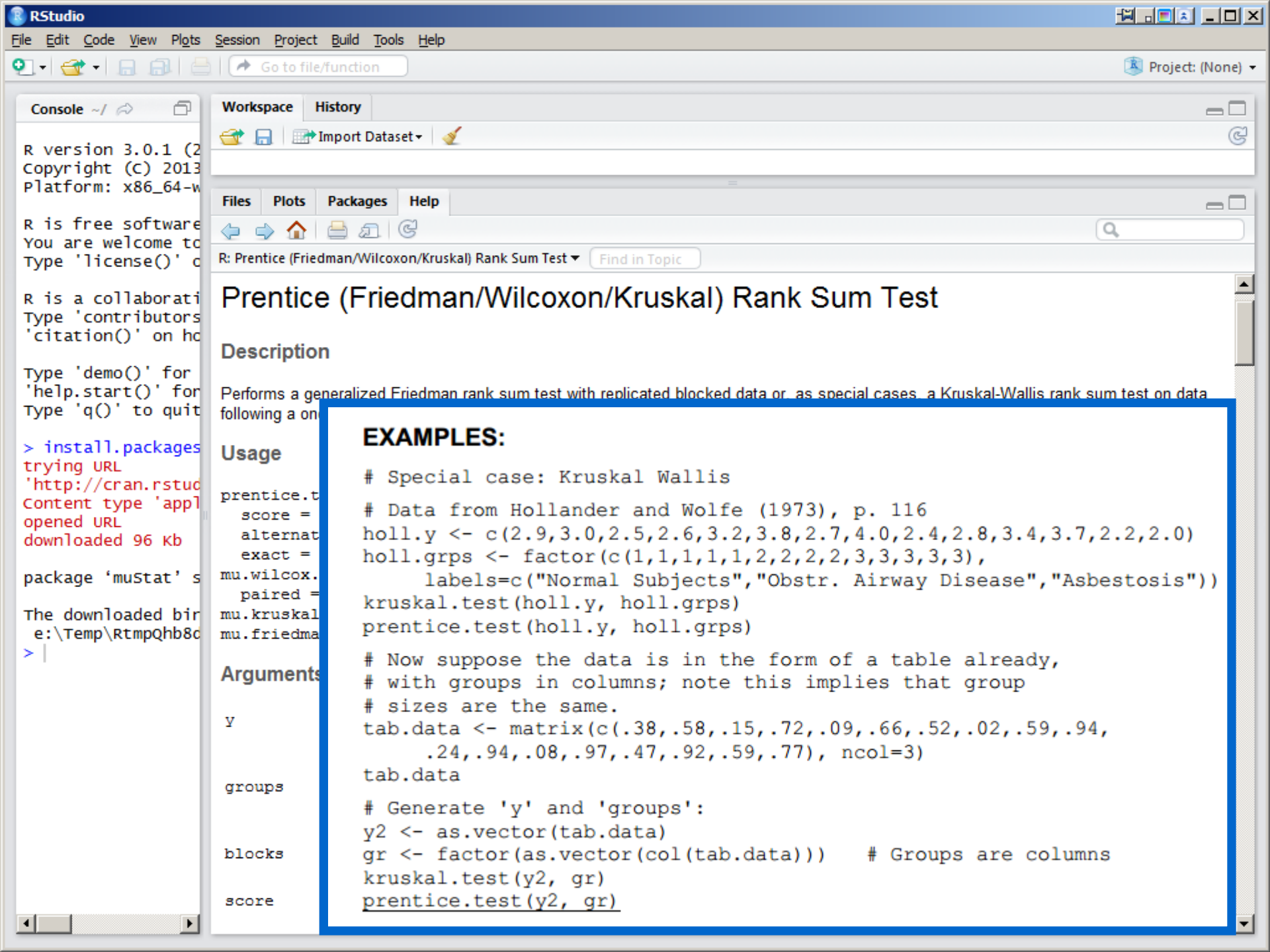
Performs a generalized Friedman rank sum test with replicated blocked data or, as special cases, a Kruskal-Wallis rank sum test on data following a one-way layout or a Wilcoxon rank sum test following a one-way layout with only two groups.

Usage

```
prentice.test(y, groups, blocks = NULL,
  score = "rank", blkwght = "prentice", condvar = TRUE,
  alternative = "two.sided", mu = 0, paired = FALSE,
  exact = NULL, correct = FALSE, df = -1, warn = 0, optim = TRUE)
mu.wilcox.test(y, groups, blocks = NULL, score = "rank",
  paired = FALSE, exact = TRUE, correct = TRUE, ...)
mu.kruskal.test(y, groups, blocks, ...)
mu.friedman.test(y, groups, blocks, ...)
```

Arguments

- y** a numeric vector of data values, NAs are used to compute block weights with incomplete block data (see `blkwght`), but will be removed with one-way designs. Blocks with observations in only one group will be removed. Infs are allowed, and are not removed as they are rankable. Required.
- groups** factor or category object of the same length as `y`, giving the group (treatment) for each corresponding element of `y`. NAs are allowed and observations with NA in `groups` will be used for scoring other observations, treating them as if they were observations in an additional (fictional) group. If not a factor or category object, it will be coerced to one. Required.
- blocks** factor or category object of the same length as `y`, giving the block membership for each corresponding element of `y`. Observations with NA in `blocks` will be removed. If not a factor or category object, it will be coerced to one.
- score** character or function object, giving the score function to be used. If NULL, `y` is assumed to already have been scored, e.g., by marginal likelihood scores (Wittkowski 1992) or u-scores (Wittkowski 2004) for multivariate data. If a function, it is



Console

```
R version 3.0.1 (2013-10-31)
Copyright (c) 2013 R Core Team
Platform: x86_64-w64-mingw32/x64

R is free software; you are free to redistribute and modify it under
the terms of the GNU General Public License. You are welcome to
Type 'license()' or 'licence()' for distribution details.

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

> install.packages("muStat")
trying URL 'http://cran.rstudio.com/web/packages/muStat/muStat_0.1-1.tar.gz'
Content type 'application/x-gzip'
opened URL
downloaded 96 kb

package 'muStat' successfully installed

The downloaded binary packages are in
e:\Temp\RtmpQhb8c\downloads

>
```

Workspace History

Import Dataset

Files Plots Packages Help

R: Prentice (Friedman/Wilcoxon/Kruskal) Rank Sum Test Find in Topic

Prentice (Friedman/Wilcoxon/Kruskal) Rank Sum Test

Description

Performs a generalized Friedman rank sum test with replicated blocked data or, as special cases, a Kruskal-Wallis rank sum test on data following a one-way ANOVA.

Usage

```
prentice.test(y, groups, blocks, score = "exact", paired = FALSE)
```

EXAMPLES:

```
# Special case: Kruskal Wallis
# Data from Hollander and Wolfe (1973), p. 116
holl.y <- c(2.9,3.0,2.5,2.6,3.2,3.8,2.7,4.0,2.4,2.8,3.4,3.7,2.2,2.0)
holl.grps <- factor(c(1,1,1,1,1,2,2,2,2,3,3,3,3,3),
  labels=c("Normal Subjects","Obstr. Airway Disease","Asbestosis"))
kruskal.test(holl.y, holl.grps)
prentice.test(holl.y, holl.grps)

# Now suppose the data is in the form of a table already,
# with groups in columns; note this implies that group
# sizes are the same.
tab.data <- matrix(c(.38,.58,.15,.72,.09,.66,.52,.02,.59,.94,
  .24,.94,.08,.97,.47,.92,.59,.77), ncol=3)
tab.data

# Generate 'y' and 'groups':
y2 <- as.vector(tab.data)
gr <- factor(as.vector(col(tab.data))) # Groups are columns
kruskal.test(y2, gr)
prentice.test(y2, gr)
```

Arguments

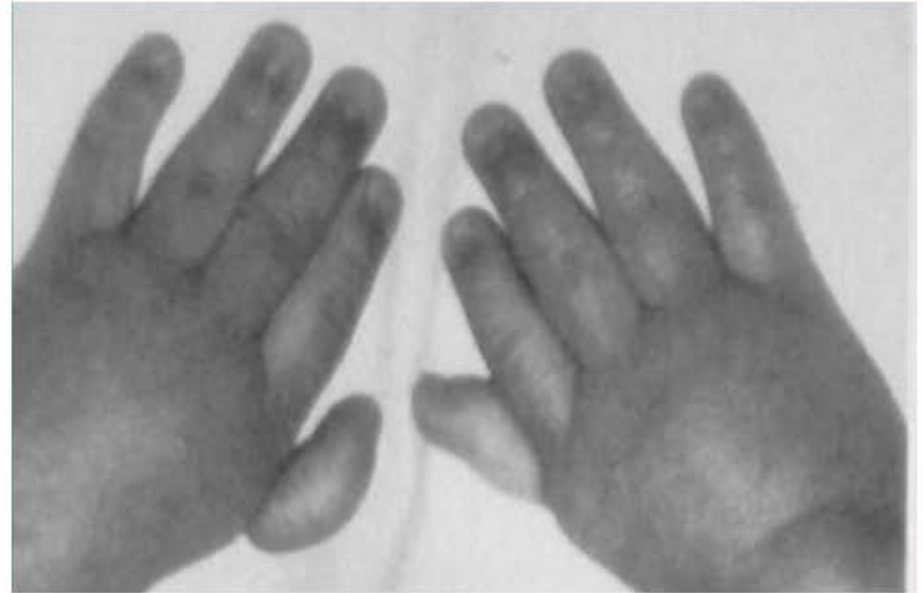
y
groups
blocks
score

So Much for Theory

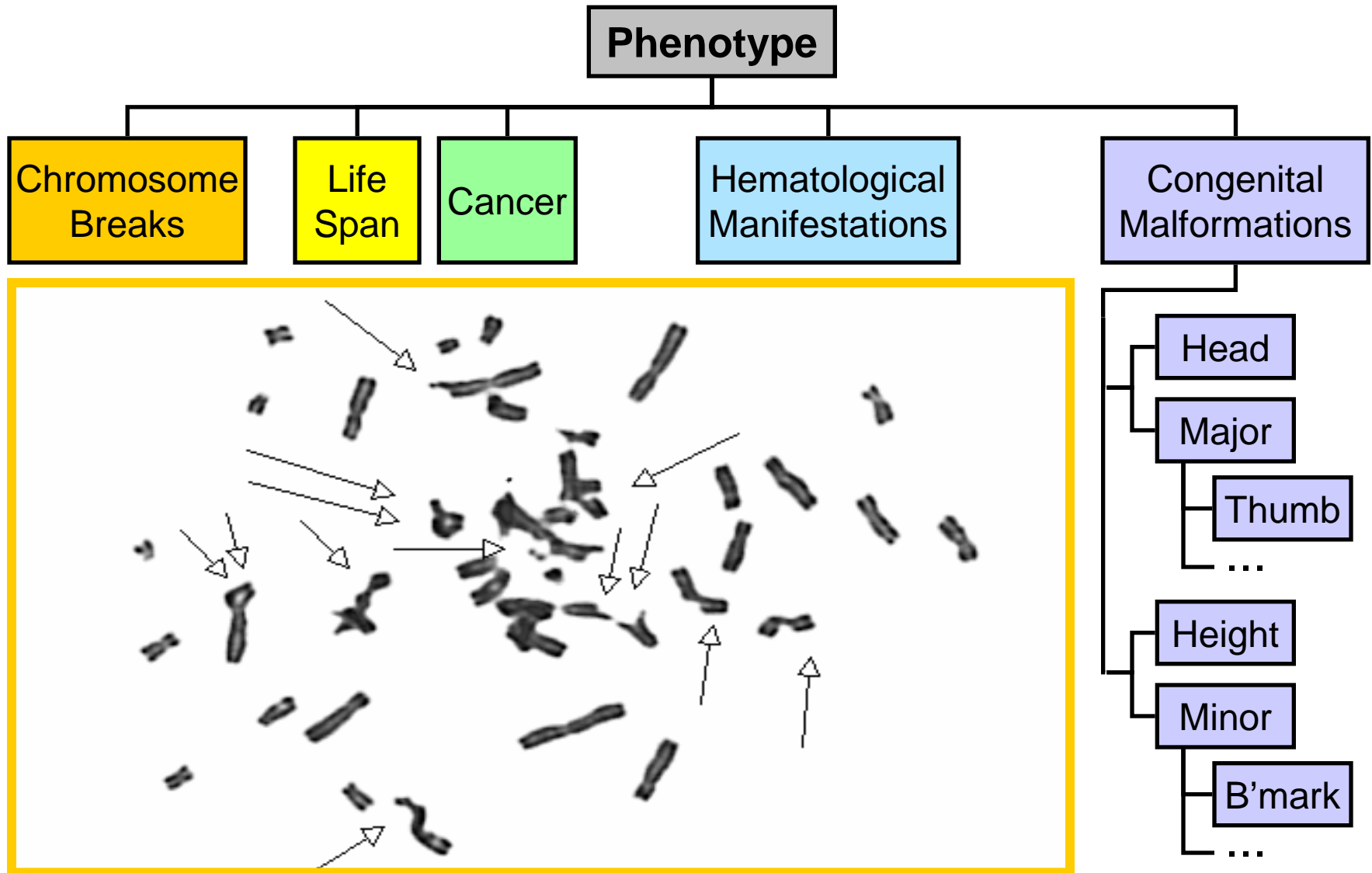


Complex Phenotypes w/censoring

Fanconi Anemia Patients

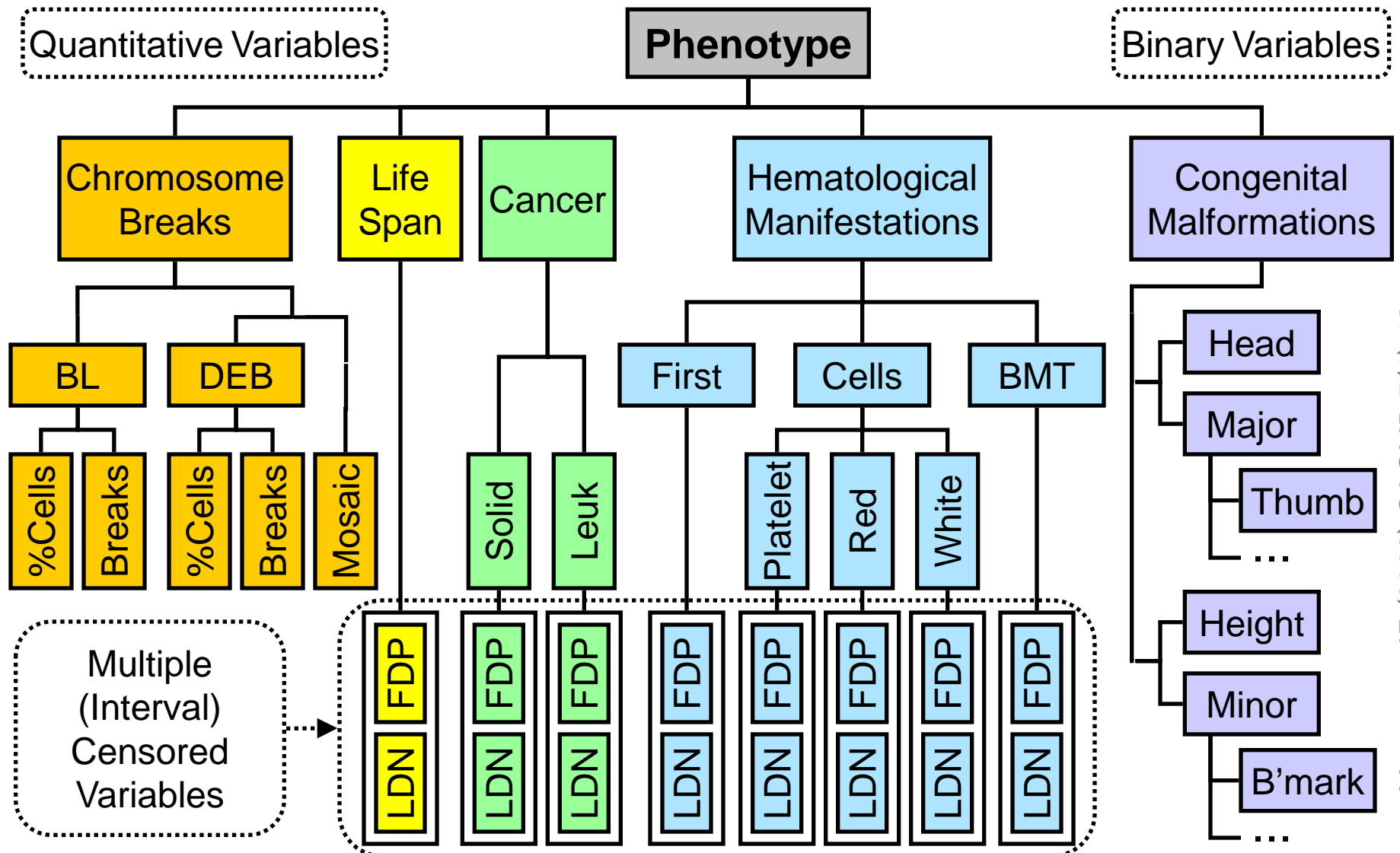


μPhene: Complex Phenotype



MORALES JF (2008) SAGMB 7(1):19

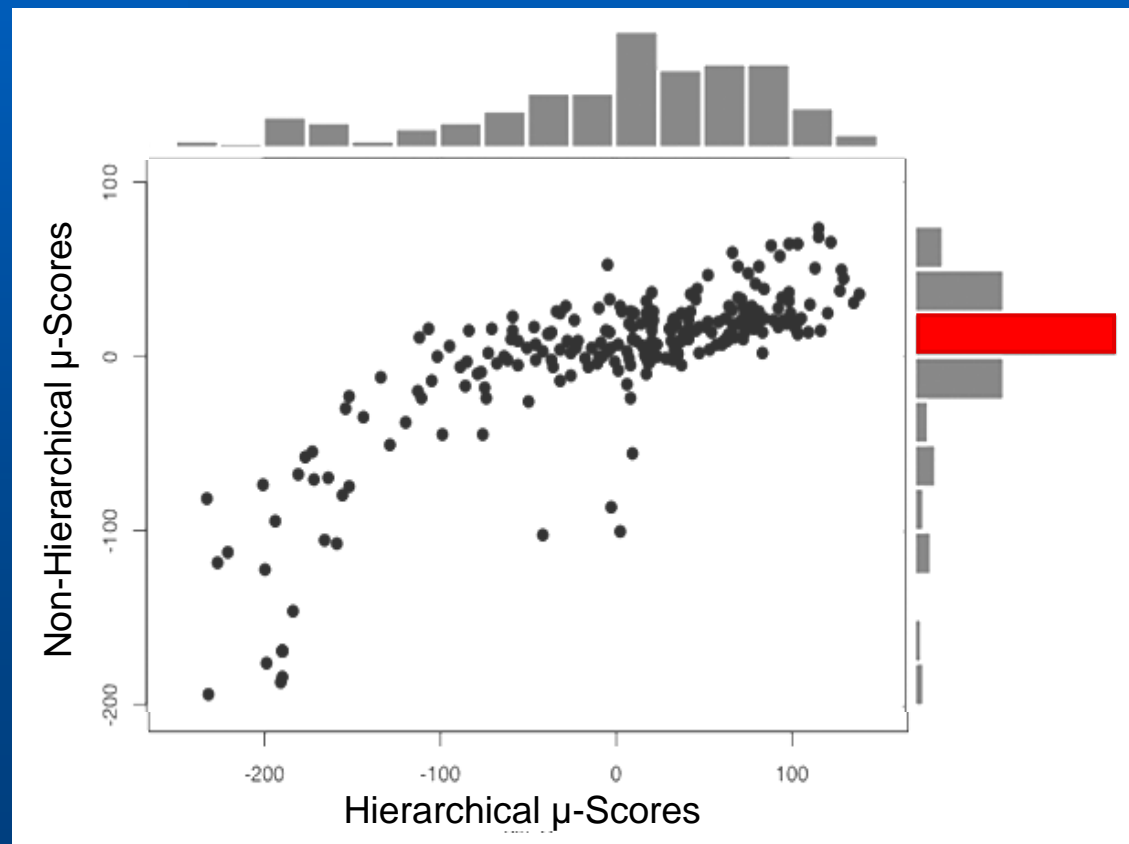
Fanconi Anemia: Complex Phenotype



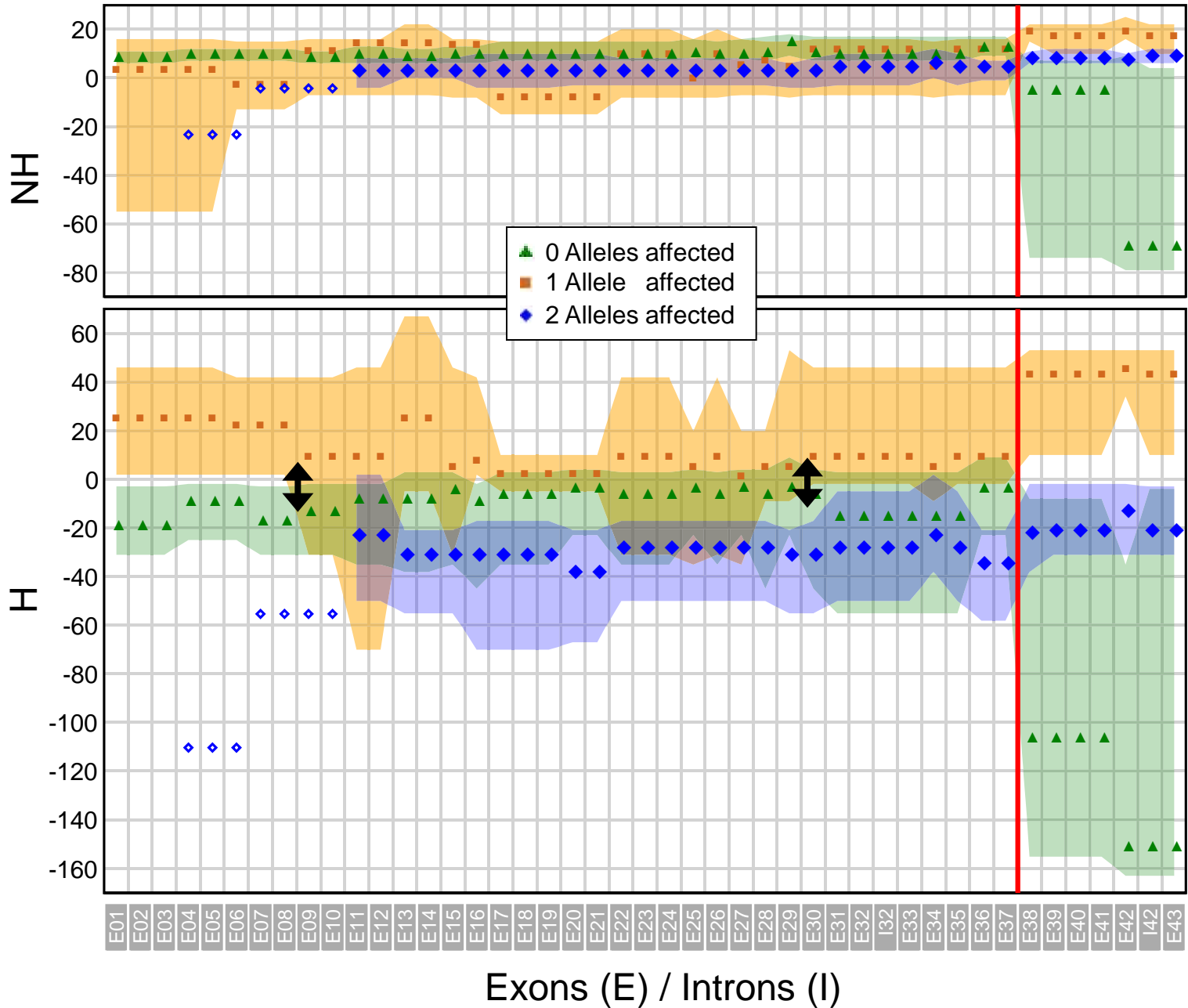
MORALES JF (2008) SAGMB 7(1):19

Advantages of Hierarchical μ -Scores

More Information Content
More Balanced Sensitivity

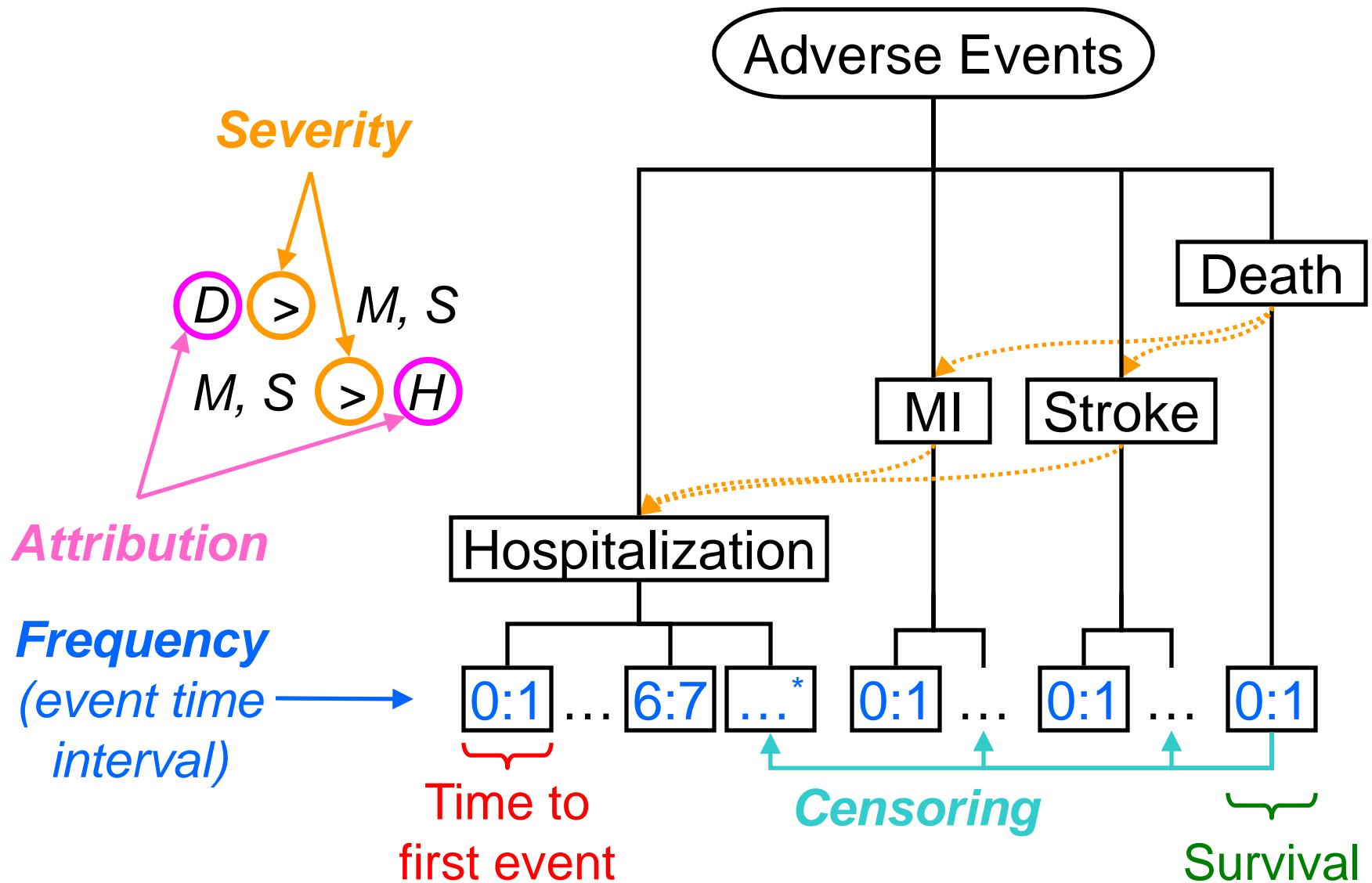


Larger Signal/Noise Ratio Better Discrimination



U-Statistics for Multivariate Data: Graded Variables

μPhene Combines Complex Outcomes



Graded Variables

Let

- **b**enign,
- **s**evere, and
- **g**rave

Graded Variables

Severity of
adverse events

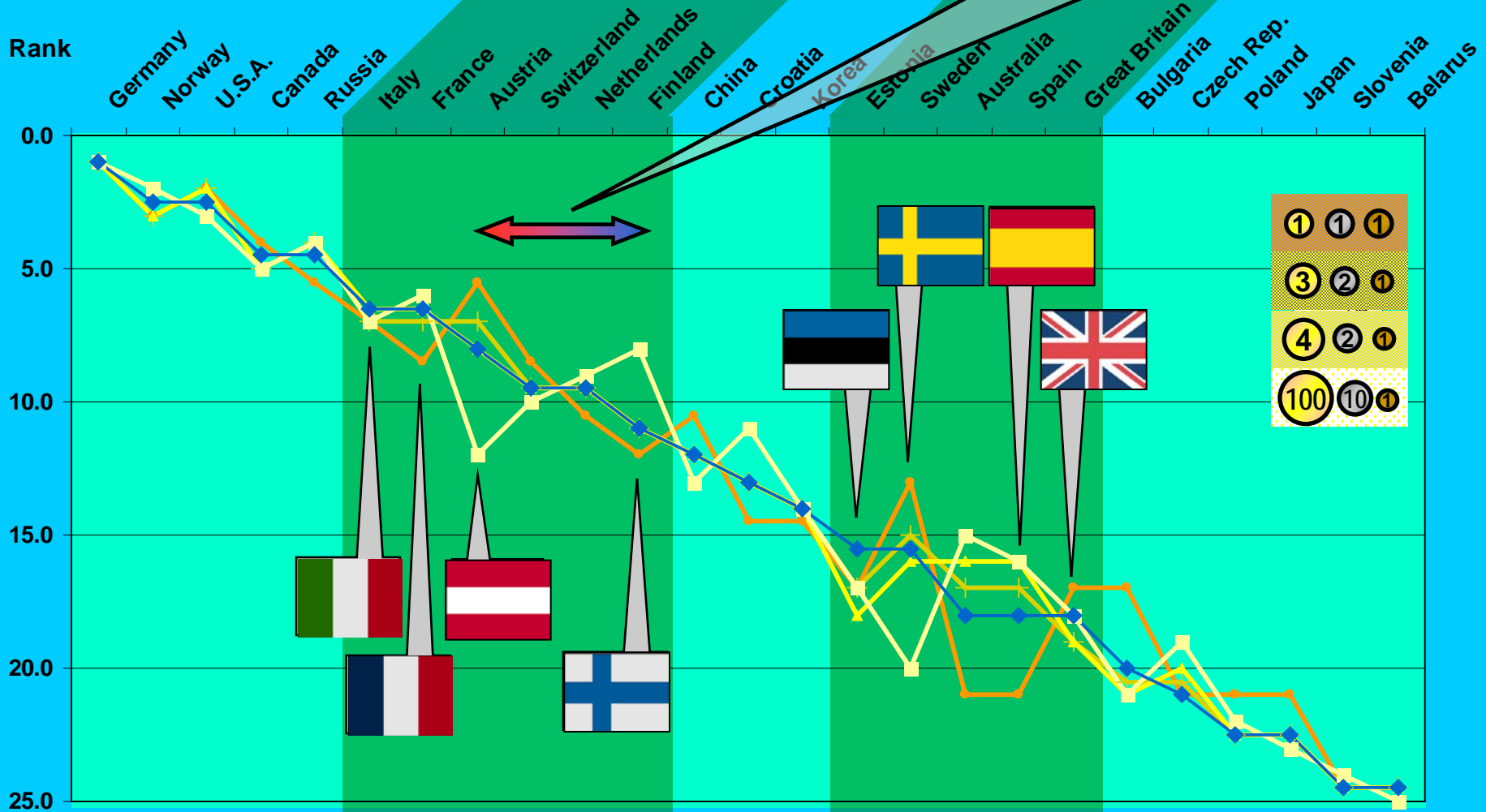
- **b**enign,
- **s**evere, and
- **g**rave

Value of
Olympic medals

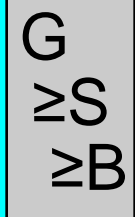
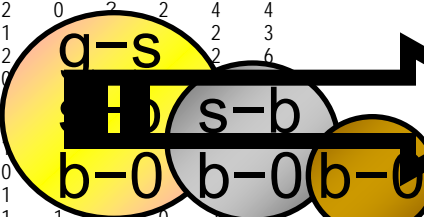
- **b**ronze,
- **s**ilver, and
- **g**old

Salt Lake City 2002

Depending on the weights
G:**S**:**B**, the rank of
Austria vs **Finland**
 can be **6:12** or **12:8**

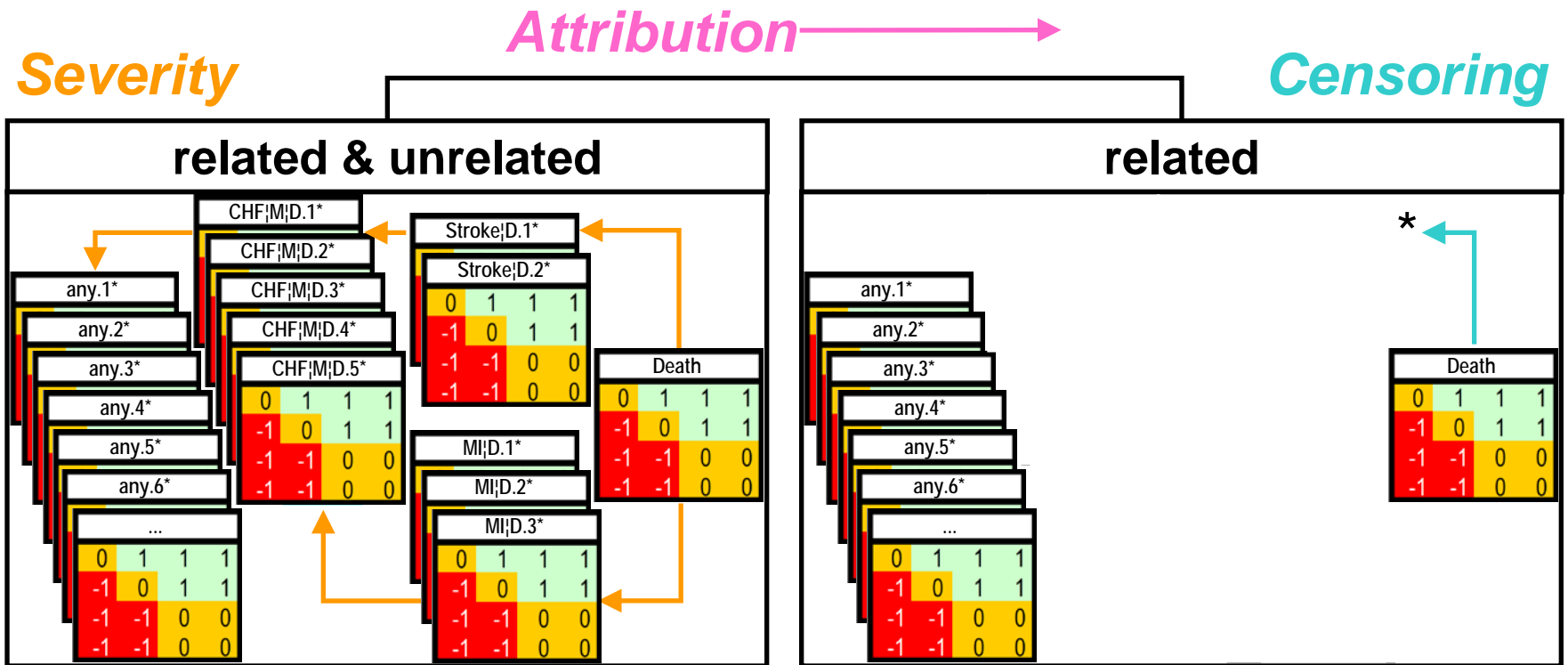


Country	g	s	b	G	$\lambda = S$	$\lambda = B$	1	1	1	0	0	0	0
Germany	12	16	7	?	12	28	50	50	44	0	0	0	0
Norway	11	7	6	?	11	18	48	46	41	-1	0	0	0
U.S.A.	10	13	11	?	10	23	46	48	50	0	0	0	0
Canada	6	3	8	?	6	9	43	34	46	0	0	-1	0
Russia	6	6	4	?	6	12	43	44	35	-1	-1	-1	0
Italy	4	4	4	?	4	8	38	37	35	-1	-1	-1	0
France	4	5	2	?	4	9	38	41	28	-1	-1	-1	0
Austria	2	4	10	?	2	6	24	37	48	0	0	-1	0
Switzerland	3	2	6	?	3	5	32	28	41	-1	-1	-1	-1
Netherlands	3	5	0	?	3	8	32	41	6	-1	-1	-1	0
Finland	4	2	1	?	4	6	38	28	18	-1	-1	-1	-1
China	2	2	4	?	2	4	24	28	35	-1	-1	-1	-1
Croatia	3	1	0	?	3	4	32	18	6	-1	-1	-1	-1
Korea	2	2	0	?	2	4	24	28	6	-1	-1	-1	-1
Estonia	1	1	1	?	1	2	16	18	18	-1	-1	-1	-1
Sweden	0	2	4	?	0	2	7	28	35	-1	-1	-1	-1
Australia	2	0	0	?	2	2	24	7	6	-1	-1	-1	-1
Spain	2	0	0	?	2	2	24	6	6	-1	-1	-1	-1
Great Britain	1	0	2	?	1	1	16	10	18	-1	-1	-1	-1
Bulgaria	0	1	2	?	0	1	7	18	28	-1	-1	-1	-1
Czech Republic	1	0	1	?	1	1	16	7	18	-1	-1	-1	-1
Poland	0	1	1	?	0	1	7	18	18	-1	-1	-1	-1
Japan	0	1	1	?	0	1	7	18	18	-1	-1	-1	-1
Slovenia	0	0	1	?	0	0	7	7	18	-1	-1	-1	-1
Belarus	0	0	1	?	0	0	7	7	18	-1	-1	-1	-1



Country	g	s	b	G	$\lambda = S$	$\lambda = B$	0	0	0	0	1	1	1
Germany	12	16	7	?	12	28	50	50	50	0	0	0	0
Norway	11	7	6	?	11	18	48	46	46	-1	0	0	0
U.S.A.	10	13	11	?	10	23	46	48	48	-1	0	0	0
Canada	6	3	8	?	6	9	43	41	44	-1	-1	-1	-1
Russia	6	6	4	?	6	12	43	44	41	-1	-1	-1	-1
Italy	4	4	4	?	4	8	38	37	38	-1	-1	-1	-1
France	4	5	2	?	4	9	38	41	35	-1	-1	-1	-1
Austria	2	4	10	?	2	6	24	33	41	-1	-1	-1	-1
Switzerland	3	2	6	?	3	5	32	30	35	-1	-1	-1	-1
Netherlands	3	5	0	?	3	8	32	37	31	-1	-1	-1	-1
Finland	4	2	1	?	4	6	38	33	28	-1	-1	-1	-1
China	2	2	4	?	2	4	24	26	31	-1	-1	-1	-1
Croatia	3	1	0	?	3	4	32	26	23	-1	-1	-1	-1
Korea	2	2	0	?	2	4	24	26	23	-1	-1	-1	-1
Estonia	1	1	1	?	1	2	16	19	18	-1	-1	-1	-1
Sweden	0	2	4	?	0	2	7	19	26	-1	-1	-1	-1
Australia	2	0	0	?	2	2	24	19	10	-1	-1	-1	-1
Spain	2	0	0	?	2	2	24	19	10	-1	-1	-1	-1
Great Britain	1	0	2	?	1	1	16	10	18	-1	-1	-1	-1
Bulgaria	0	1	2	?	0	1	7	10	18	-1	-1	-1	-1
Czech Republic	1	0	1	?	1	1	16	10	10	-1	-1	-1	-1
Poland	0	1	1	?	0	1	7	10	10	-1	-1	-1	-1
Japan	0	1	1	?	0	1	7	10	10	-1	-1	-1	-1
Slovenia	0	0	1	?	0	0	7	3	3	-1	-1	-1	-1
Belarus	0	0	1	?	0	0	7	3	3	-1	-1	-1	-1

μPhene Data Structure



Frequency
(event time interval)

Welcome, **kmw** [Change Password](#)

RU BERD Tools

- [μGWAS@Home](#)
- [μNews](#)
- [Log Out](#)
- [Contact](#)
- [Help](#)

μStat

- [Start New Job](#)
- [List Jobs](#)
- [Utilities](#)

WISDOM

- [BERD DashBoard](#)
- [Study Design/Form Design](#)






REDCap

- [Export to REDCap and Epi Info](#)
- [Import from REDCap](#)
- [Open REDCap](#)

Internal

- [Script Manager](#)
- [List All Jobs](#)
- [Server Stats](#)
- [Clean Up](#)

Start a New Job

	<input type="button" value="Cancel"/> <input type="button" value="Start"/>
Analysis Type 	<input checked="" type="radio"/> Scoring <input type="radio"/> Testing
Version 	<input type="radio"/> N411 (SVN 839) <input checked="" type="radio"/> N826 (SVN 937) <input type="radio"/> N911 (SVN 966)
Device Group 	CPU <input type="button" value="v"/>
Parameter File 	<input type="button" value="Choose File"/> No file chosen
Data/Zip File 	<input type="button" value="Choose File"/> No file chosen
Notes	<div style="border: 1px solid gray; height: 100px;"></div>

No Parameter file for Scoring Jobs

Data Preparation: Cum. Variables

days until (death)

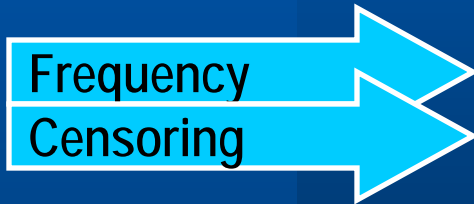
days until 1st of (any stroke | death)
 days until 2nd of (any stroke | death)

days until 1st of (any MI | death)
 days until 2nd of (any MI | death)
 days until 3rd of (any MI | death)

days until 1st of (any CHF | stroke | MI | death)
 days until 2nd of (any CHF | stroke | MI | death)
 . . .
 days until nth of (any CHF | stroke | MI | death)

days until 1st of (any other | CHF | stroke | MI | death)
 days until 2nd of (any other | CHF | stroke | MI | death)
 . . .
 days until nth of (any other | CHF | stroke | MI | death)

μPhene Data Structure



Hdr2	Death	Death/Stroke				Death/Myocardial Infarction					Death/Stroke/MI/CHF					Any Complications			
Pht 1	1																		
Pht 2	1	2						3			4								
Pht 3	1		2																
Pht 4	1	1	2	1	2	3	1			n	1				1			n	
Ivl		-1	1	-2	2	-3	3	-4	4	-5	5
Pol	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PID																			
Dat 1																			
Dat 2																			
Dat 3																			
Dat 4																			
...																			

μPhene Data Structure

Attribution

Grading

Frequency
Censoring

Hdr 1	Related Complications ¹															All Complications		
Hdr 2	Death	Death/Stroke			Death/Myocardial Infarction			Death/Stroke/MI/CHF			Any Complications			...				
Pht 1	1															2		
Pht 2	1	2						3			4							
Pht 3		1			2													
Pht 4	1	1	2	1	2	3	1		n	1			n					
Ivl		-1	1	-2	2	-3	3	-4	4	-5	5	
Pol	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
PID																		
Dat 1																		
Dat 2																		
Dat 3																		
Dat 4																		
...																		

Combining Endpoint Evidence

With u-scores for multivariate ordinal data (μ -Scores), one can objectively score

- Multiple
- Graded
- Repeated
- Censored Events.

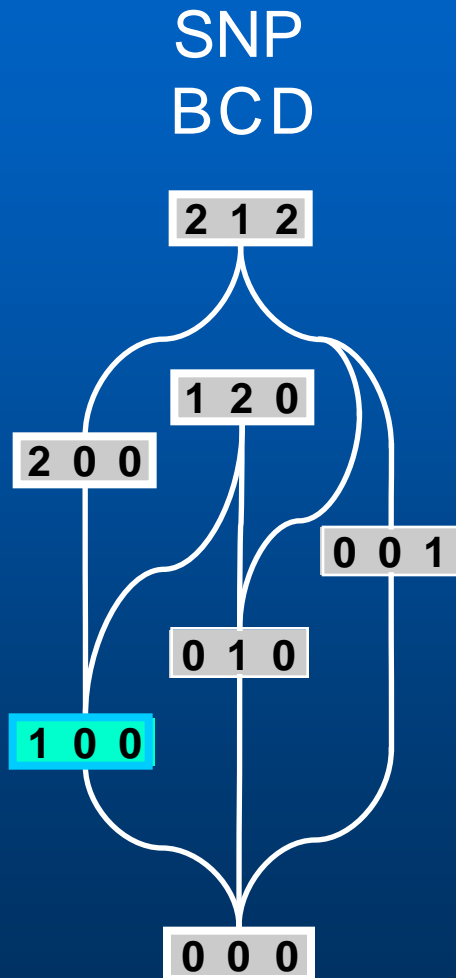
2011:
 μ -scores
accepted
by FDA for
Phase III

2009: What are the remaining problems?

- Sufficiently detailed data remains elusive as long as
- Regulatory agencies are satisfied with crude the measures
 - Time to first event
 - Overall survival

U-Statistics for Multivariate Data: GWAS

More than one SNP



SNP.B

SNP.C

SNP.D

0:

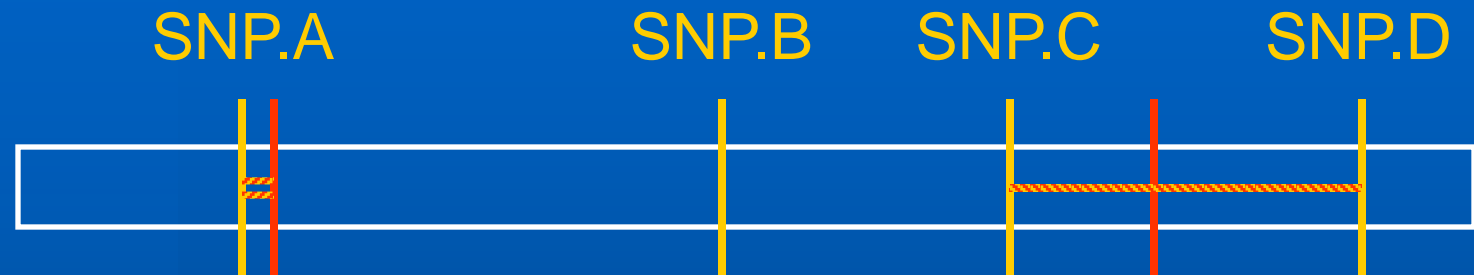
1:

 X

2:

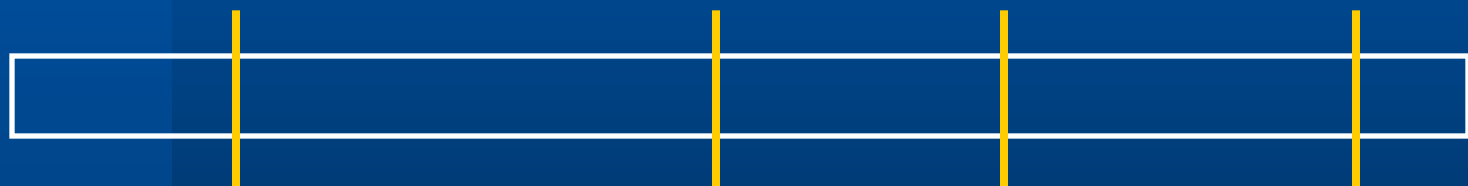
 X
X

Linkage Disequilibrium (LD)



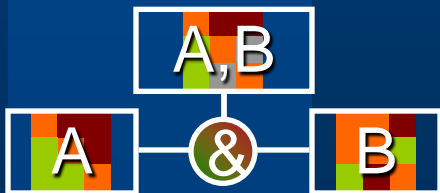
Disease Locus ?
LD primarily with A

Disease Locus ?
LD with C and D



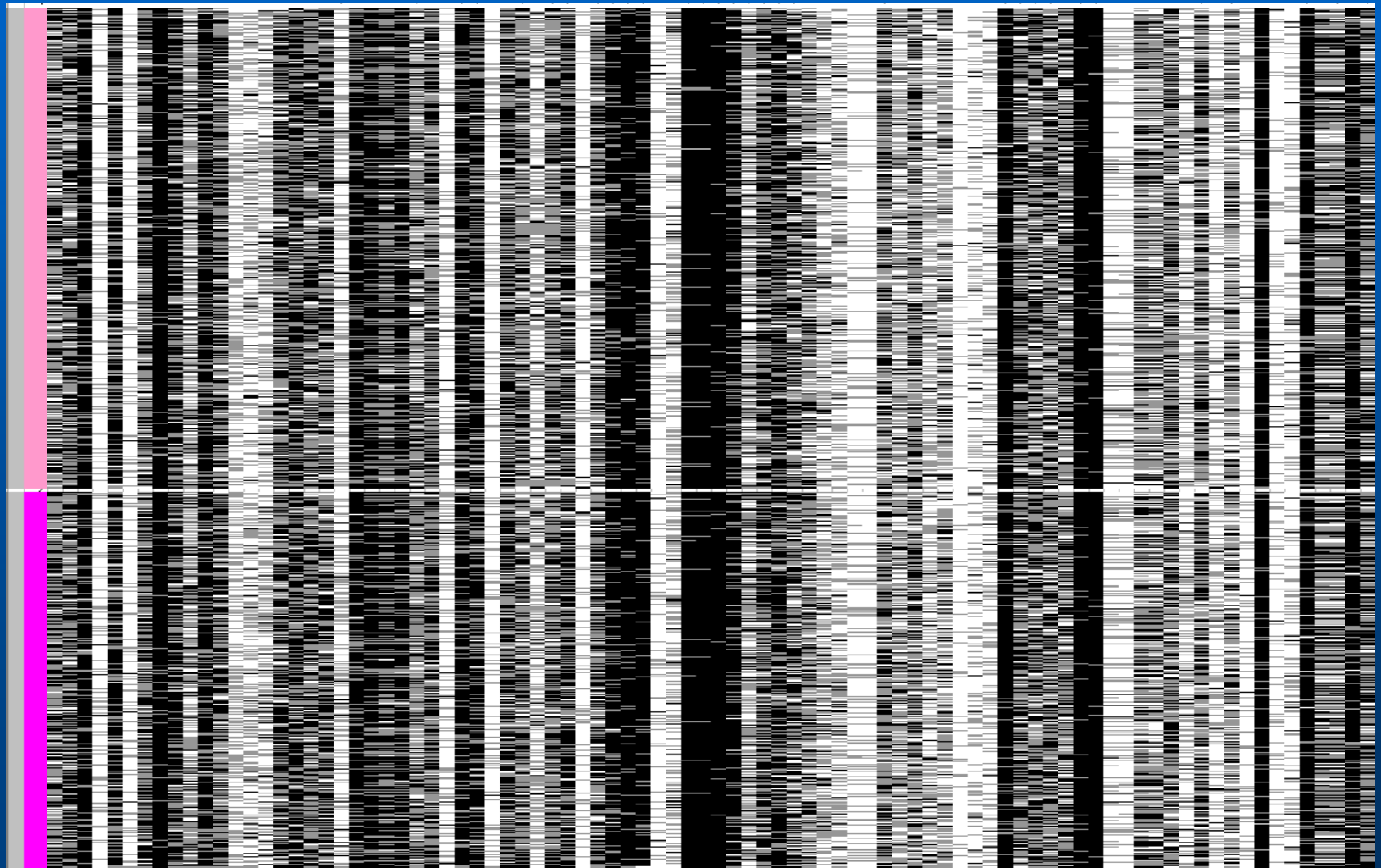
A (A, B) B (B, C) C (C, D) D

Analysis of Genetic Intervals

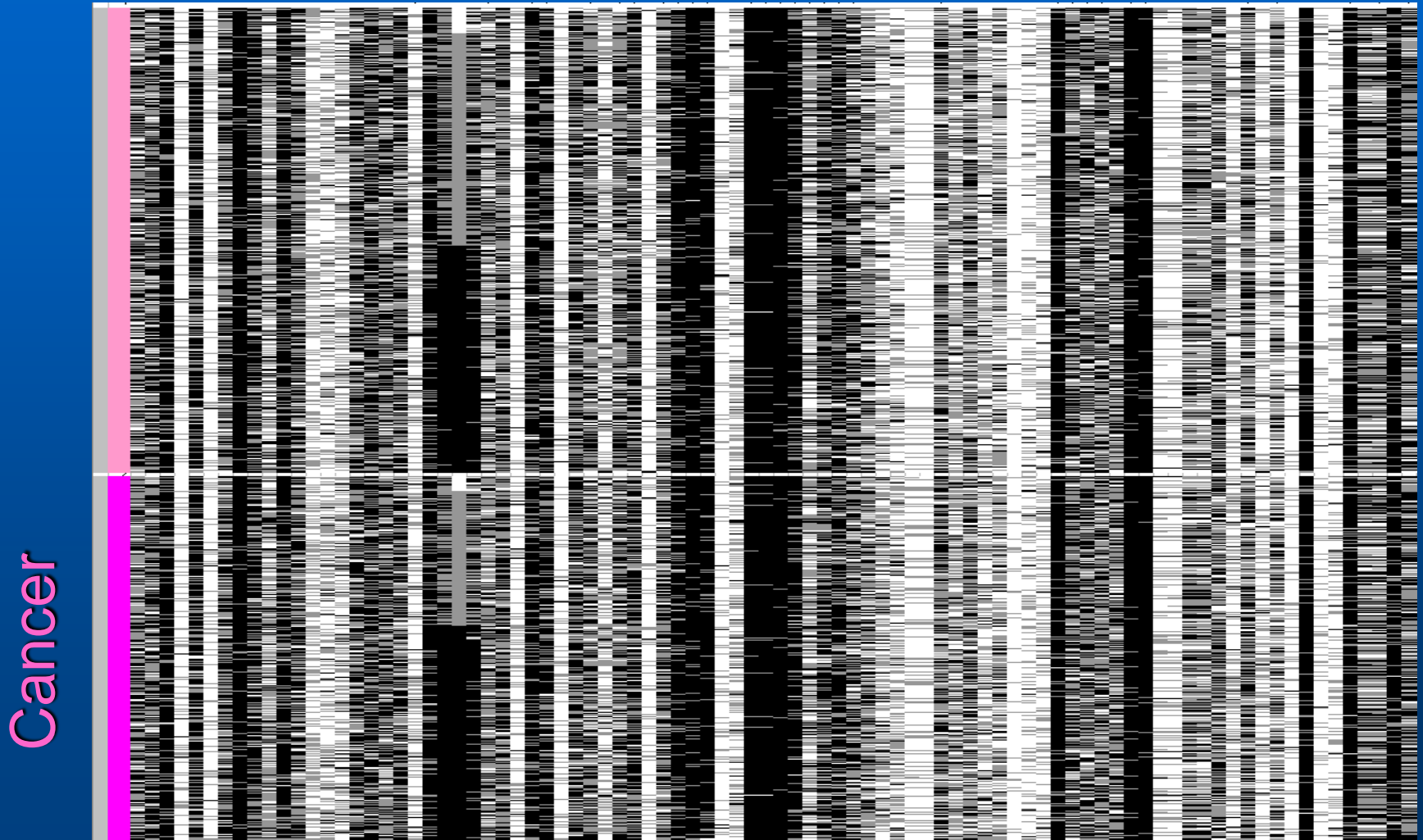


SNP Data sorted by ID

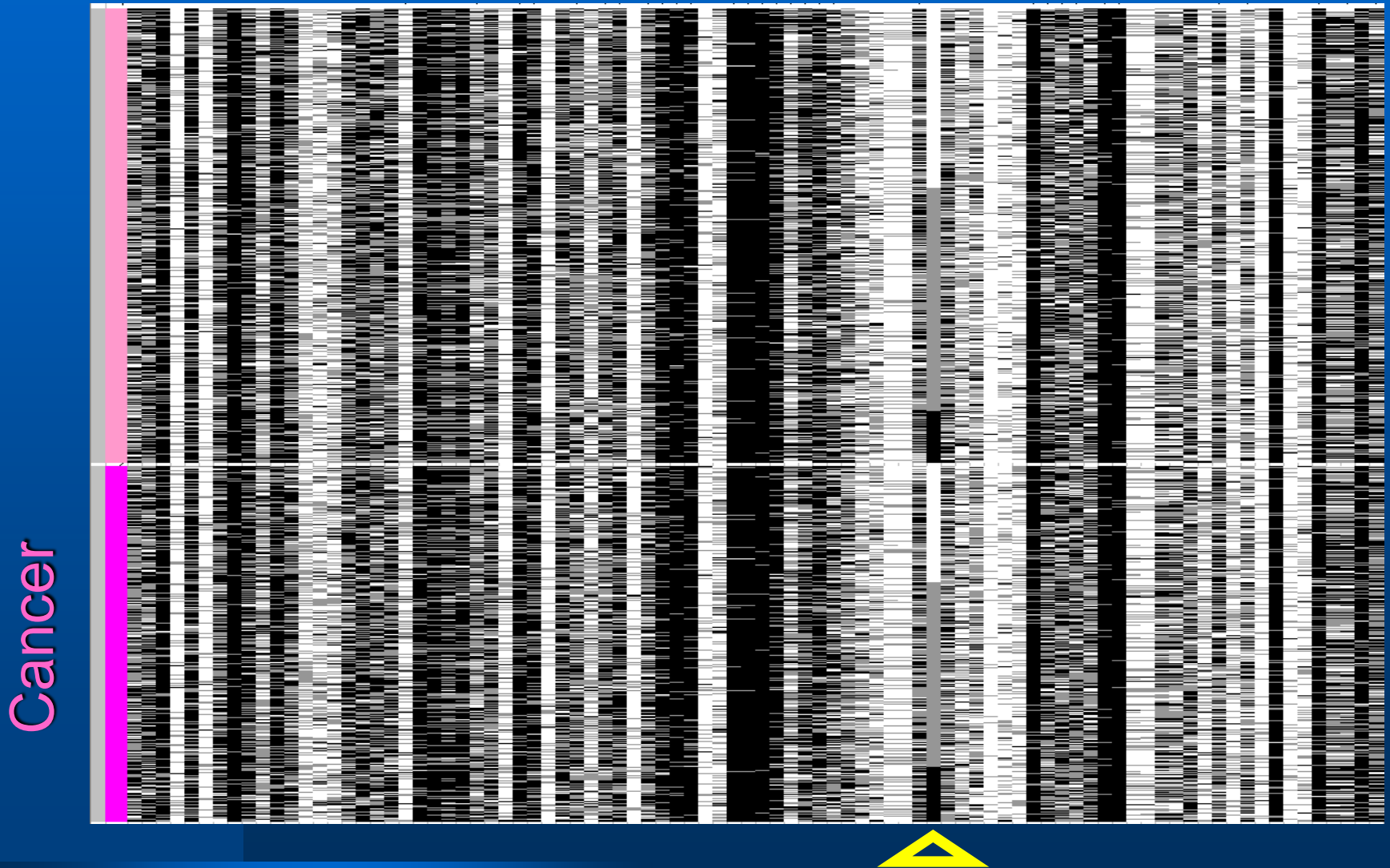
Cancer



SNP Data Sorted by 1st signif SNP

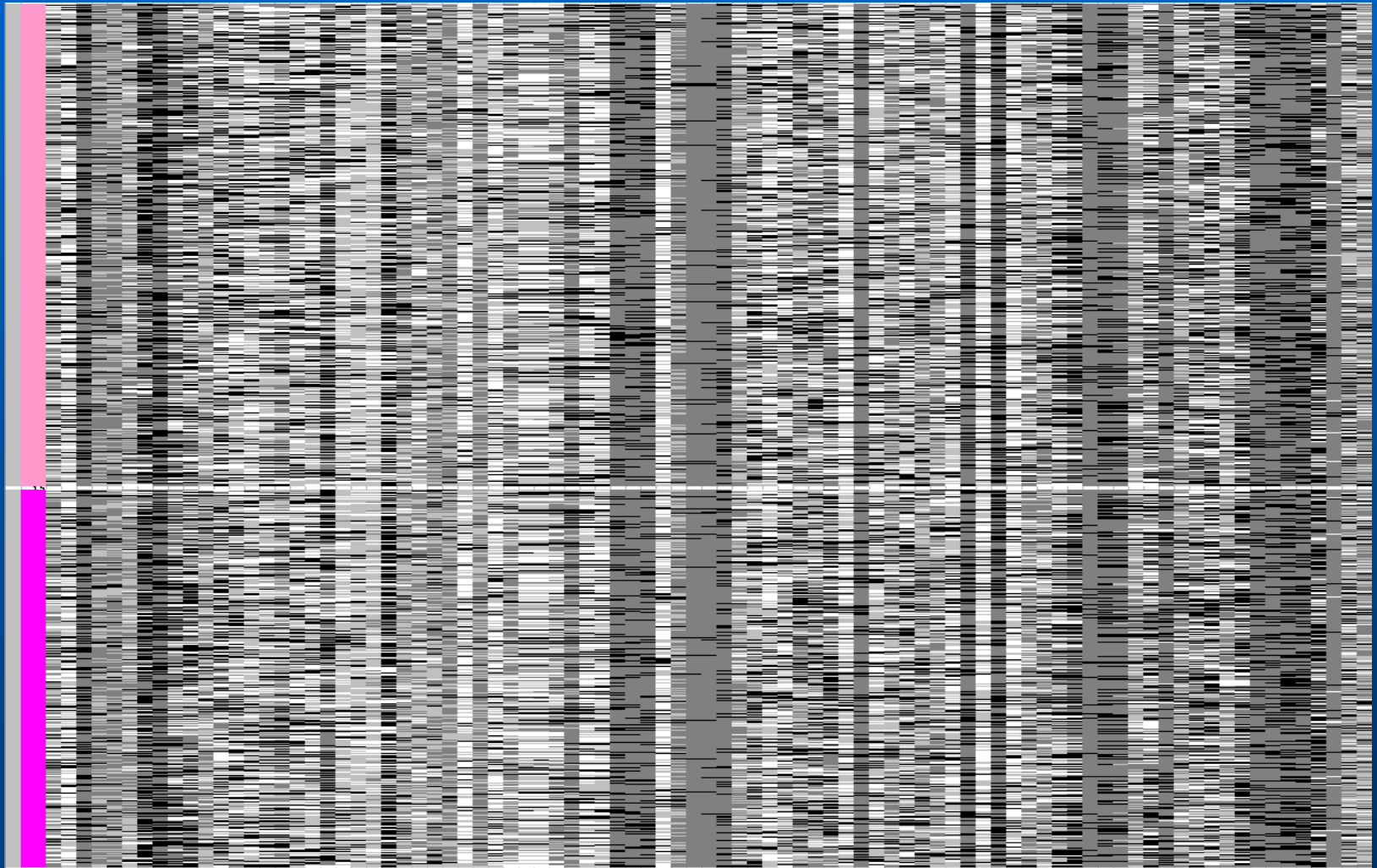


SNP Data Sorted by 2nd signif SNP

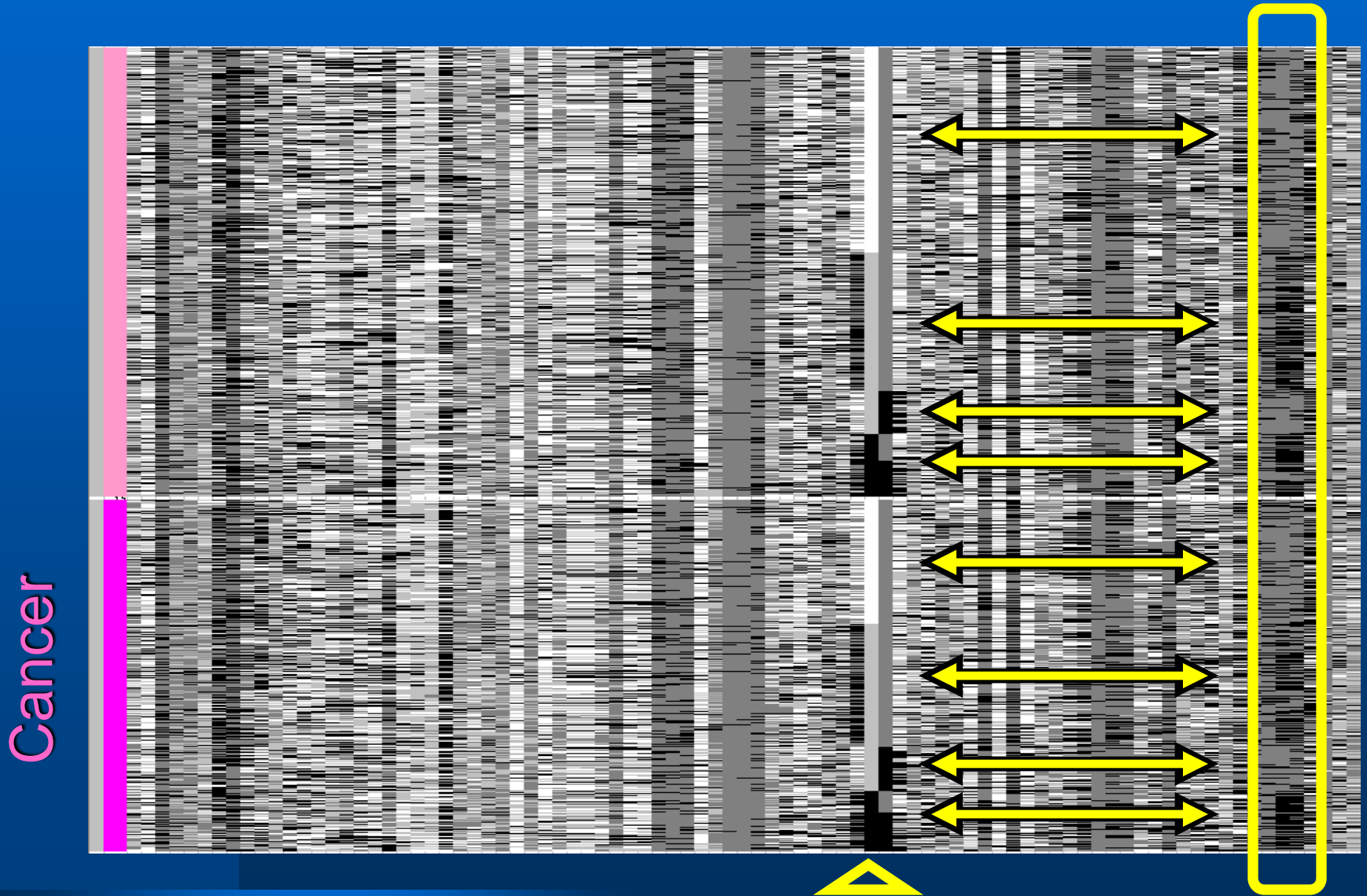


Interval Scores Sorted by ID

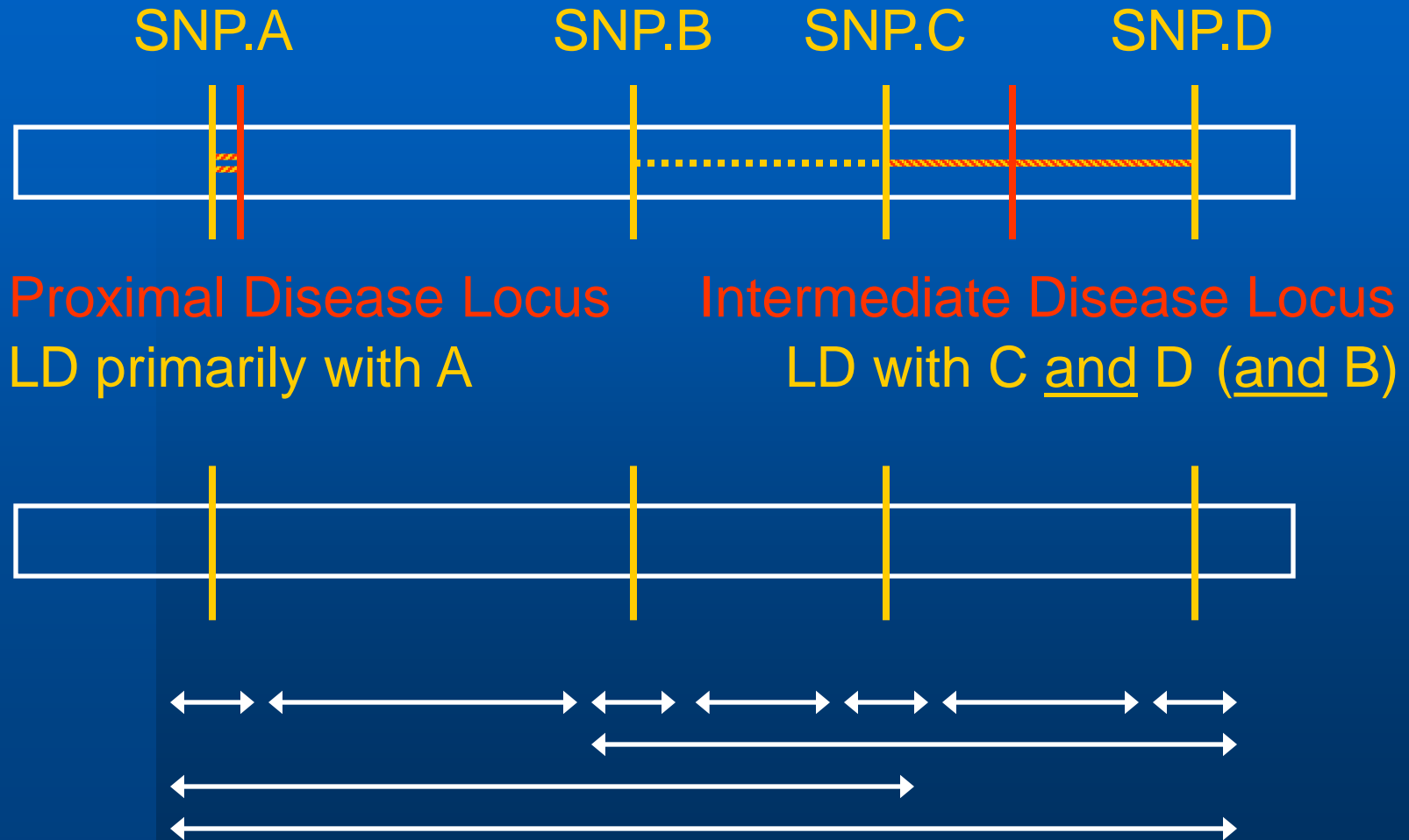
Cancer



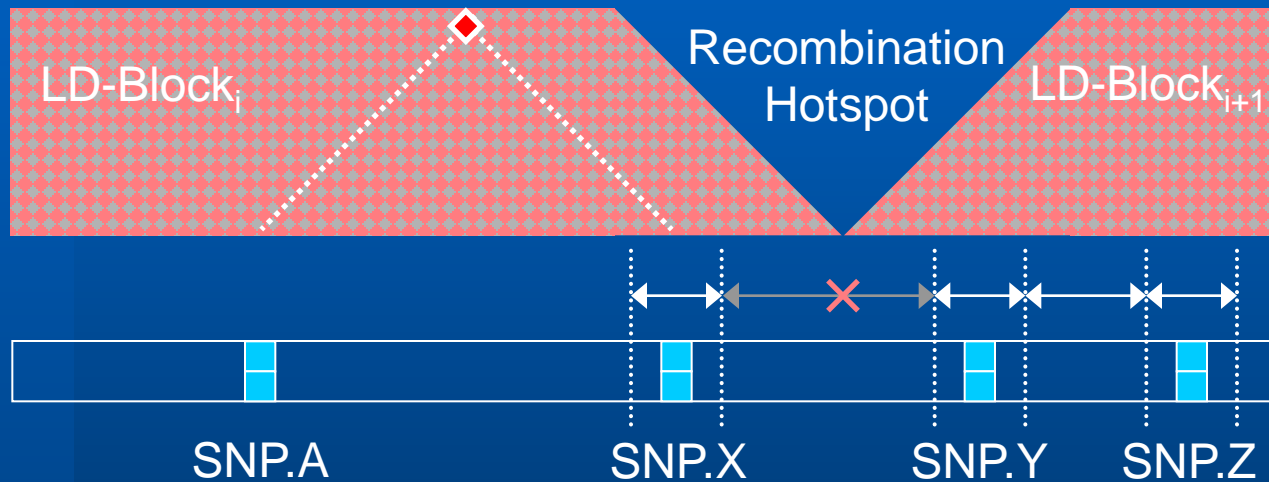
Interval Scores by 1st signif Interval

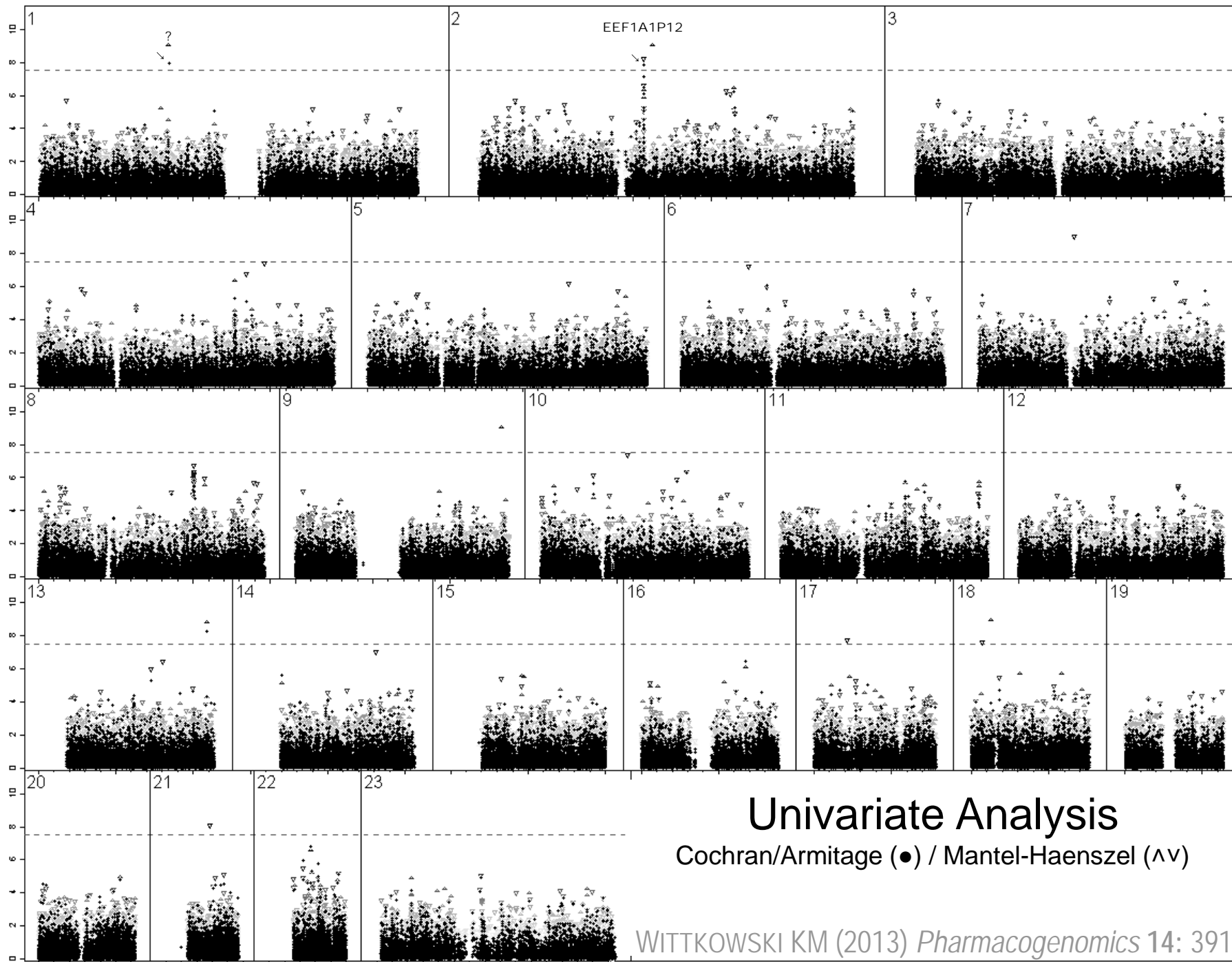


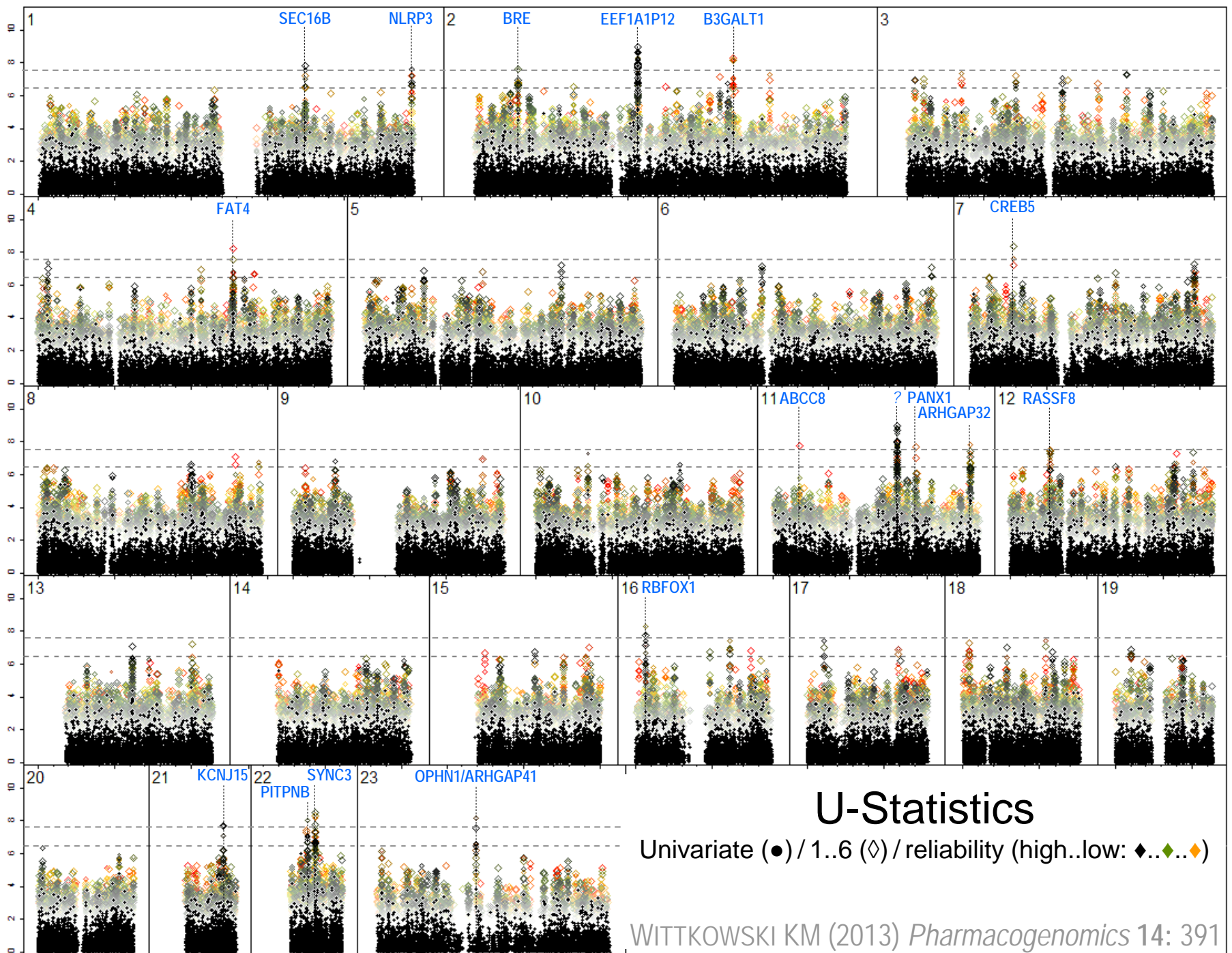
Extended Disequilibrium



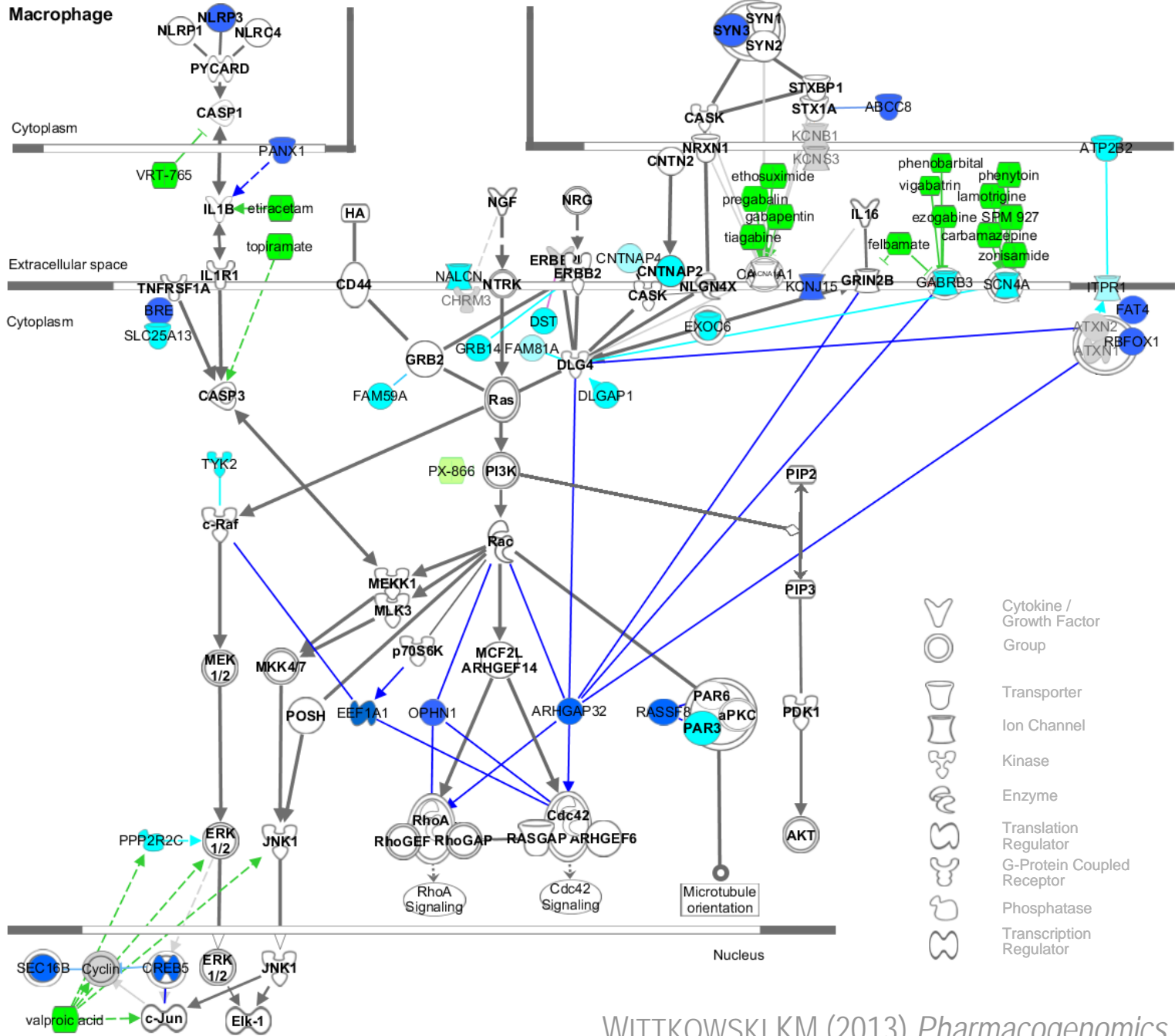
Linkage Disequilibrium (LD)







Macrophage

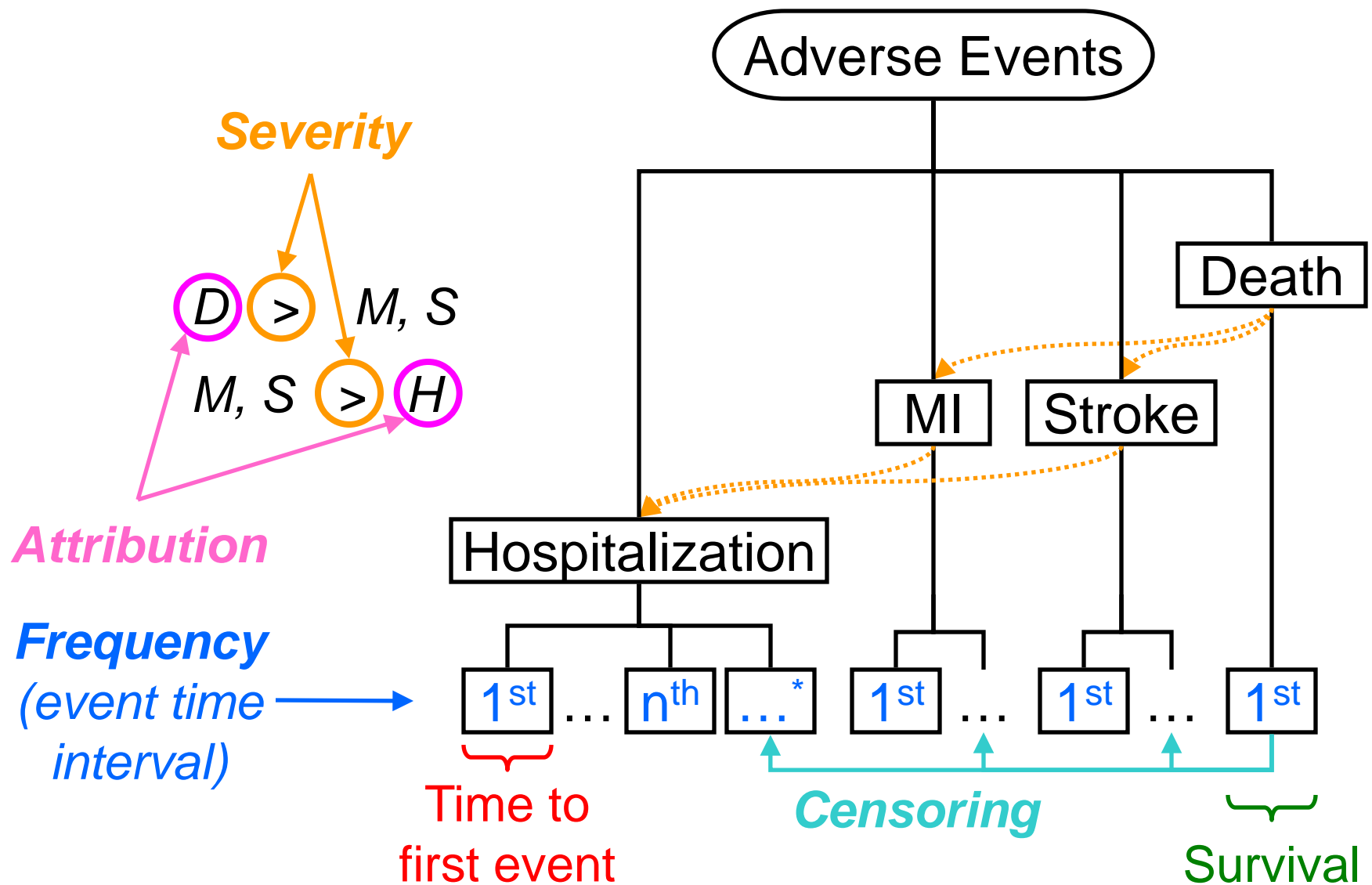


Putting it all together:

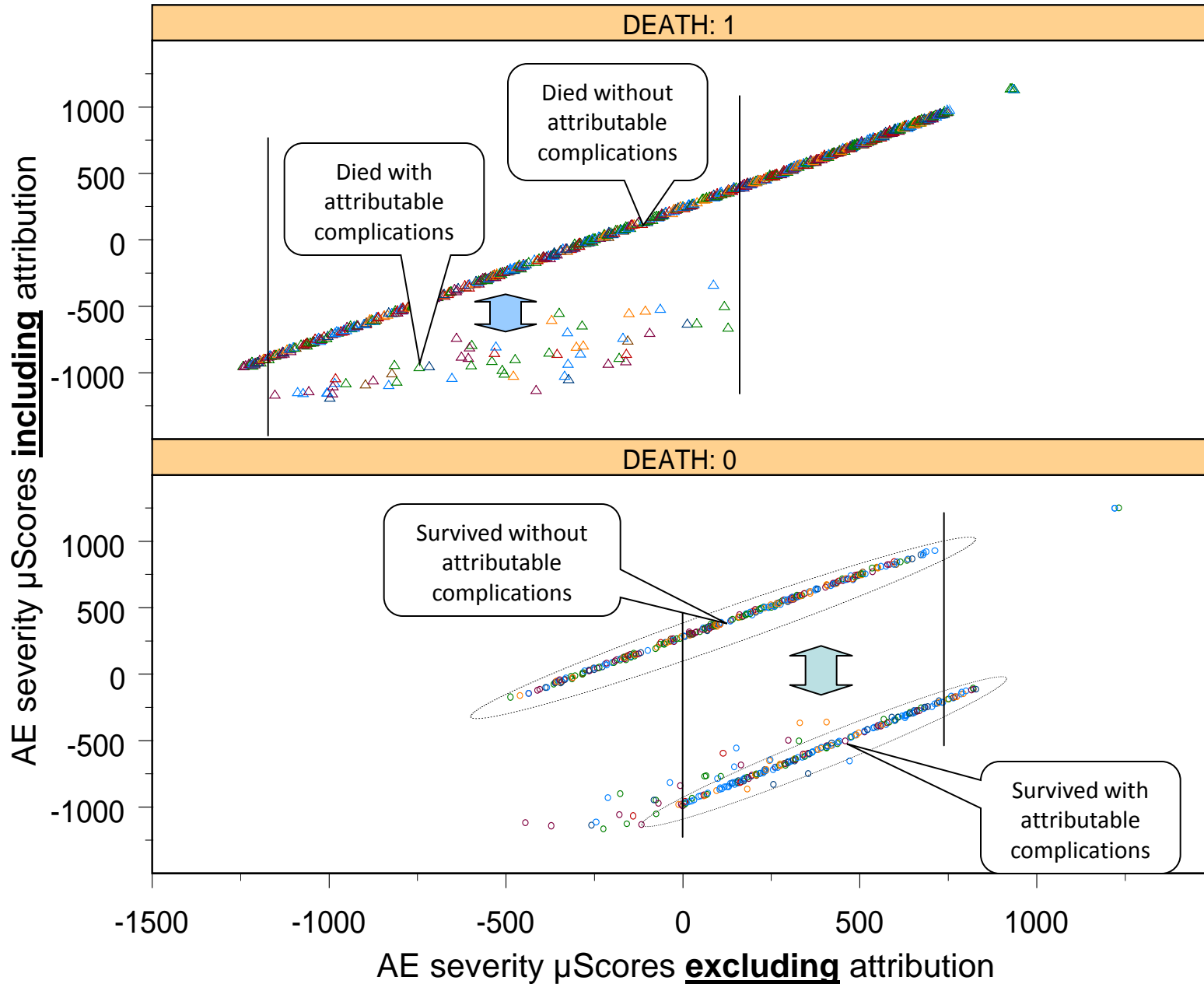
“Gold mining” Phase III Data

- Non-Response
- Adverse Events

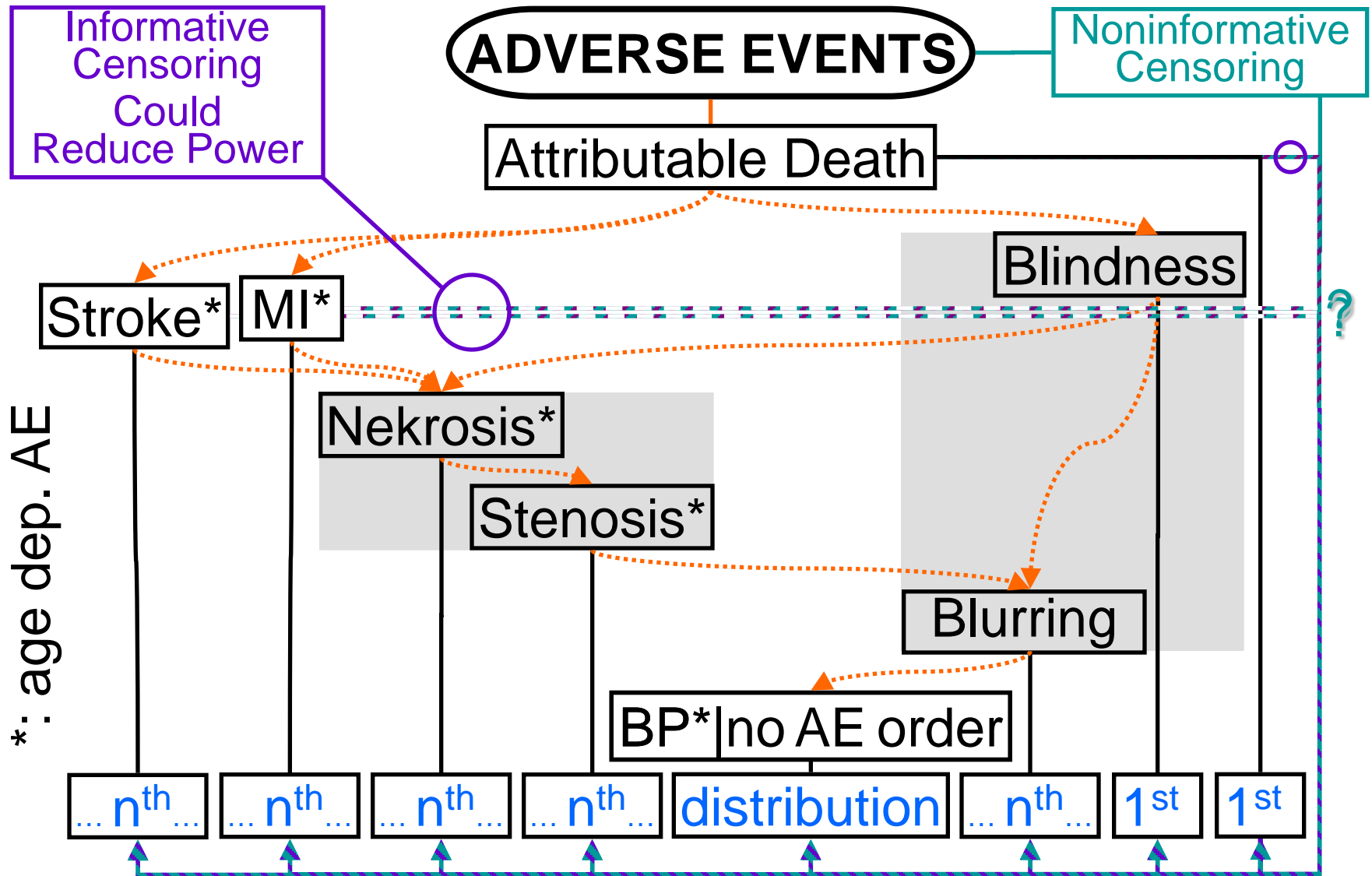
μPhene Combines Complex Outcomes



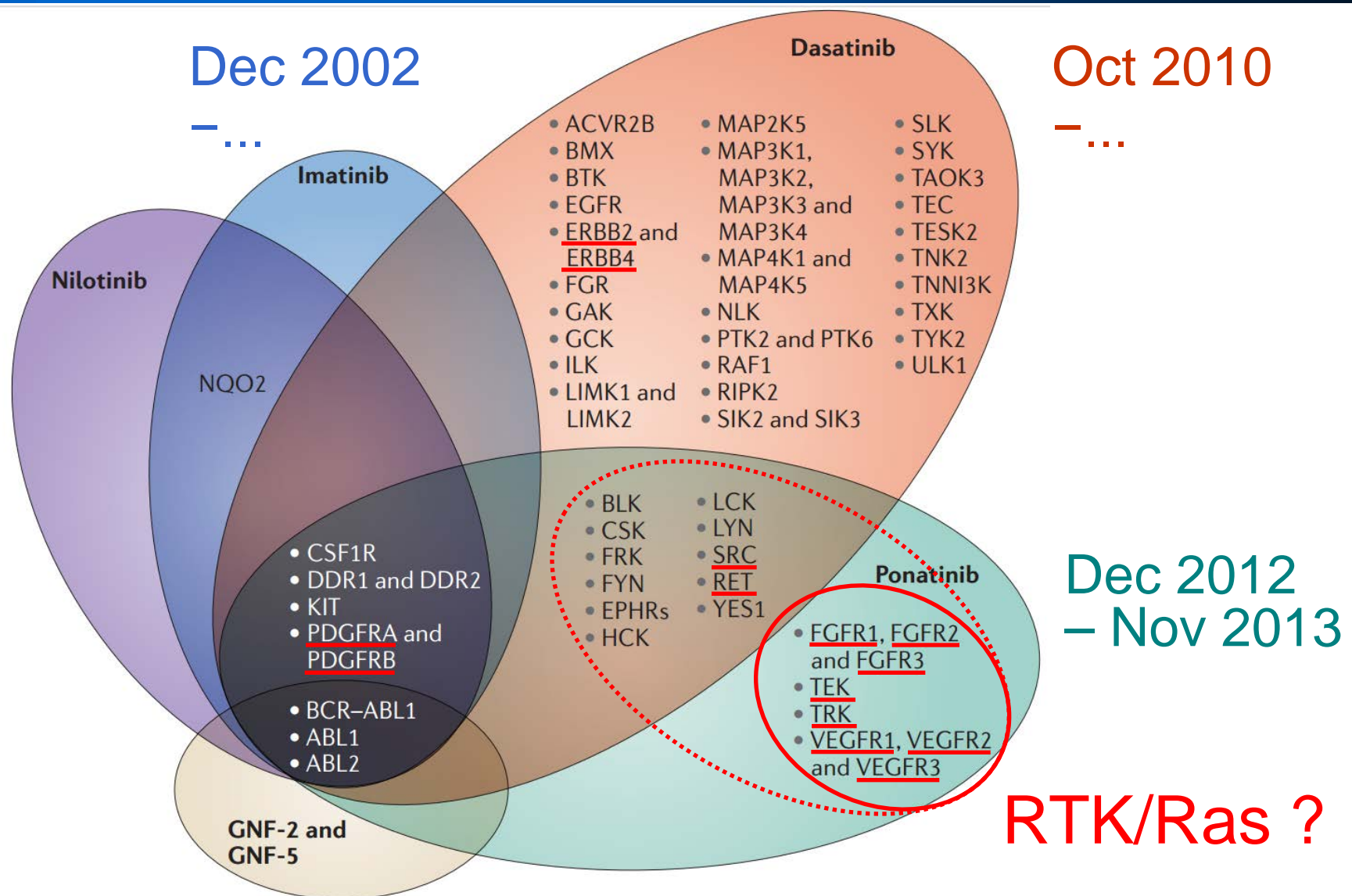
“Gold-mining” Phase III Data



μPhene Combines Complex Outcomes

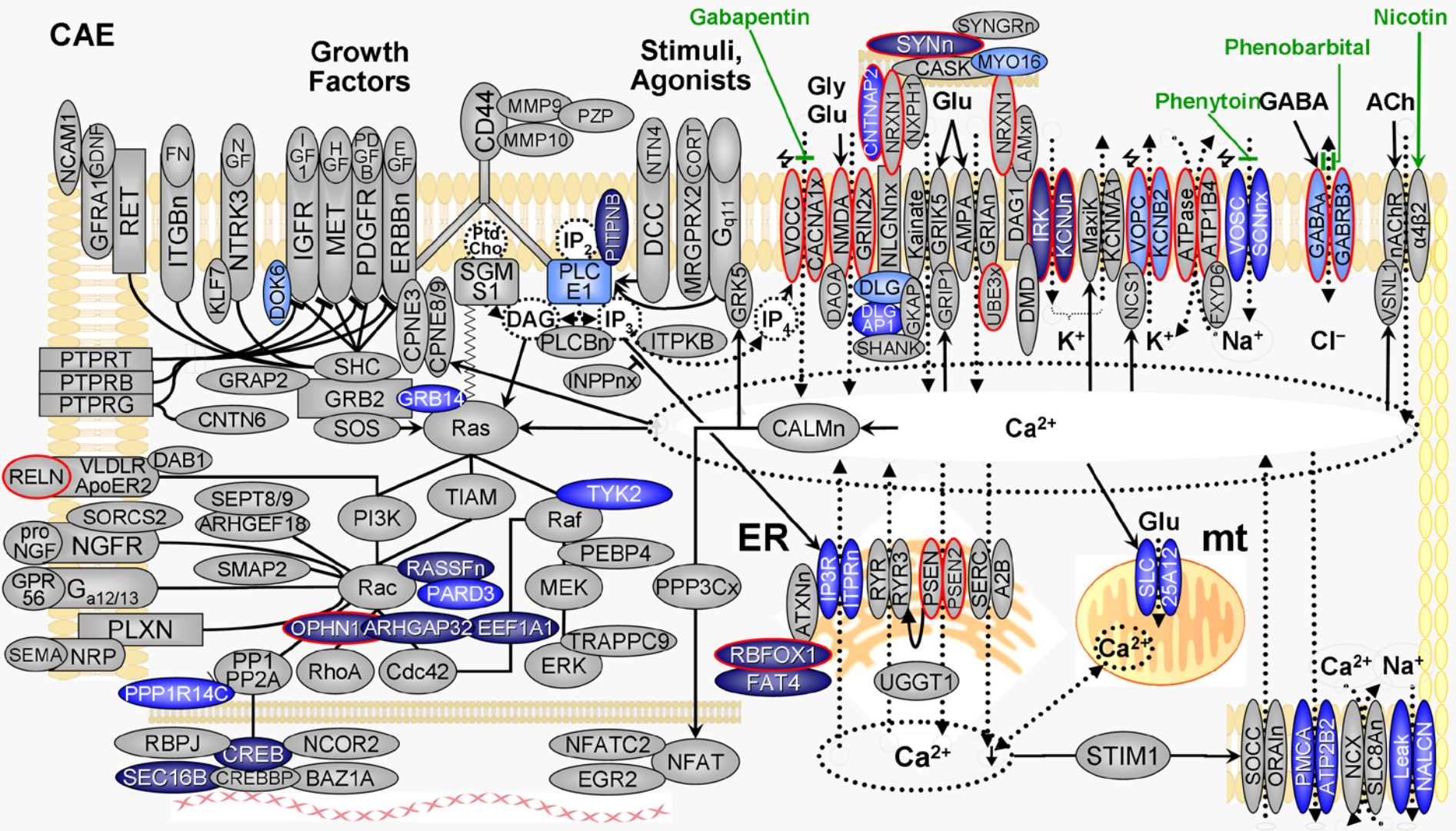


Targets of BCR-ABL Inhibitors

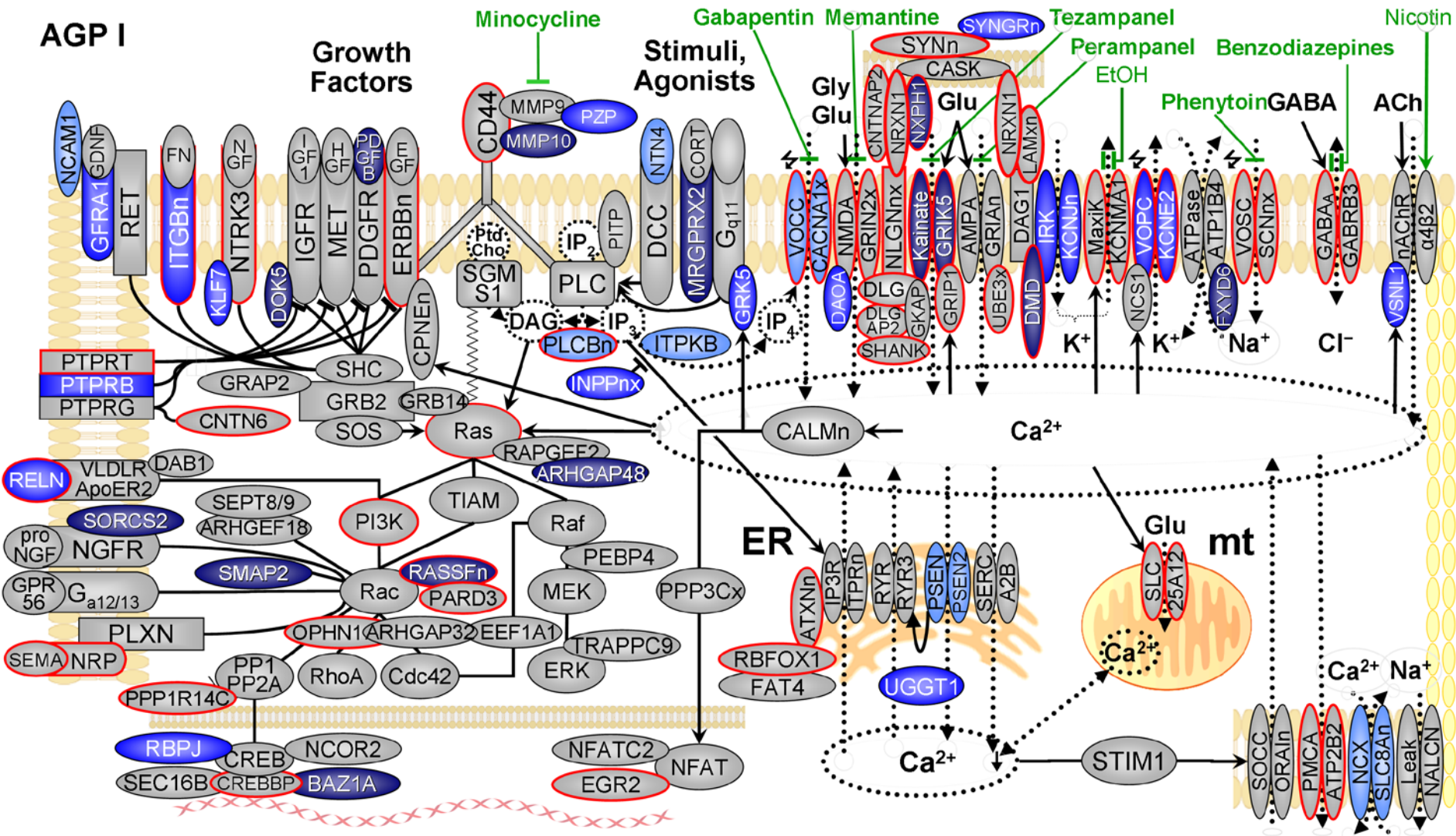


Results:

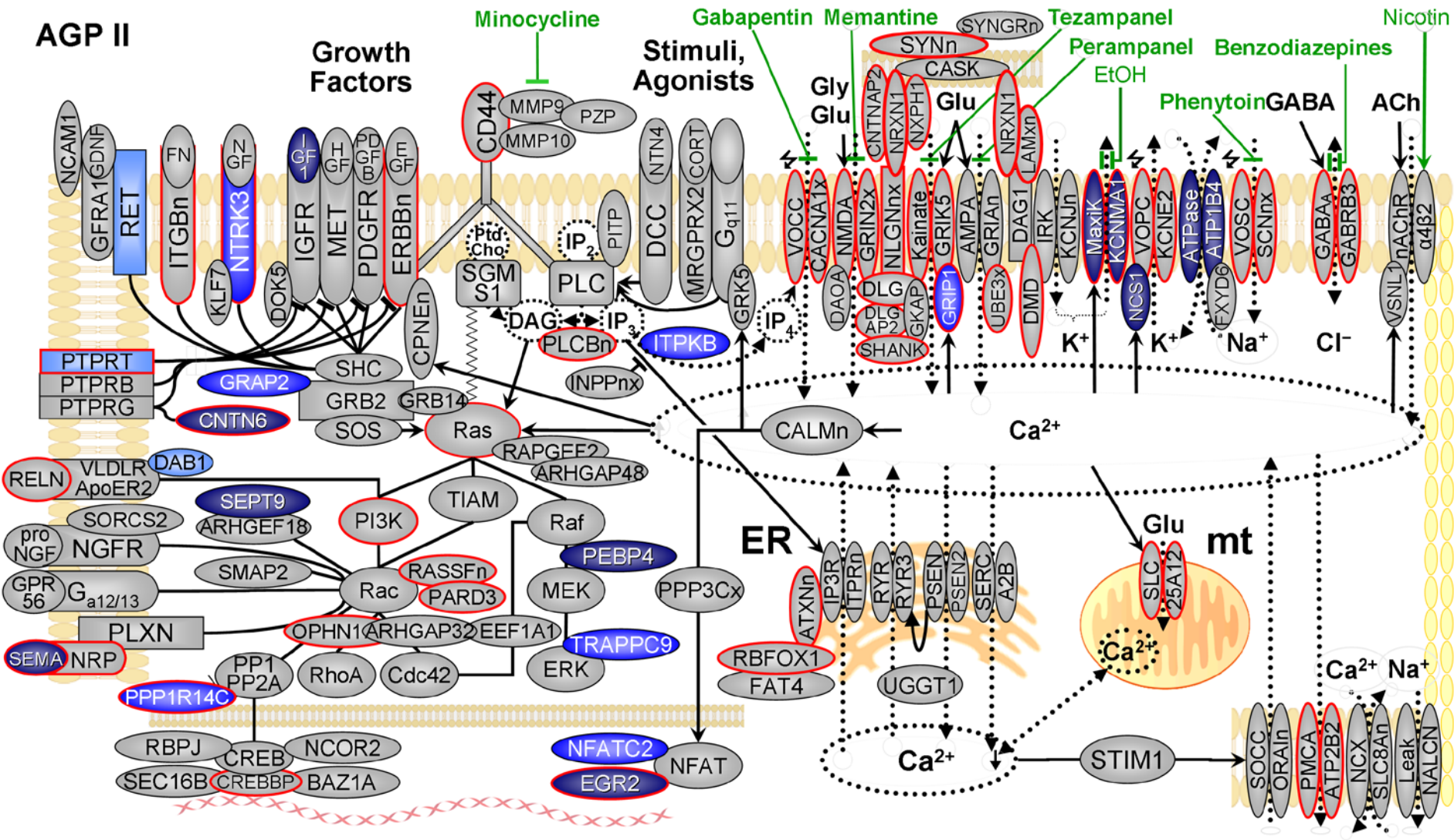
Ras pathway (Autism GWAS)

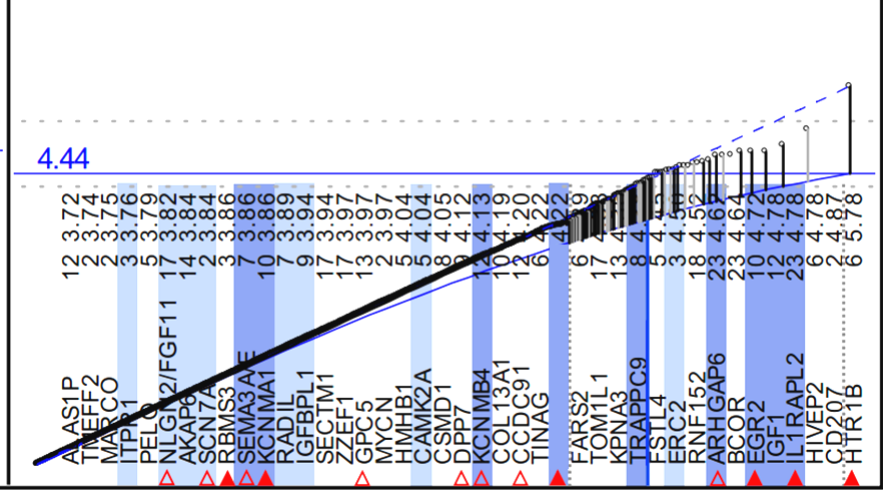
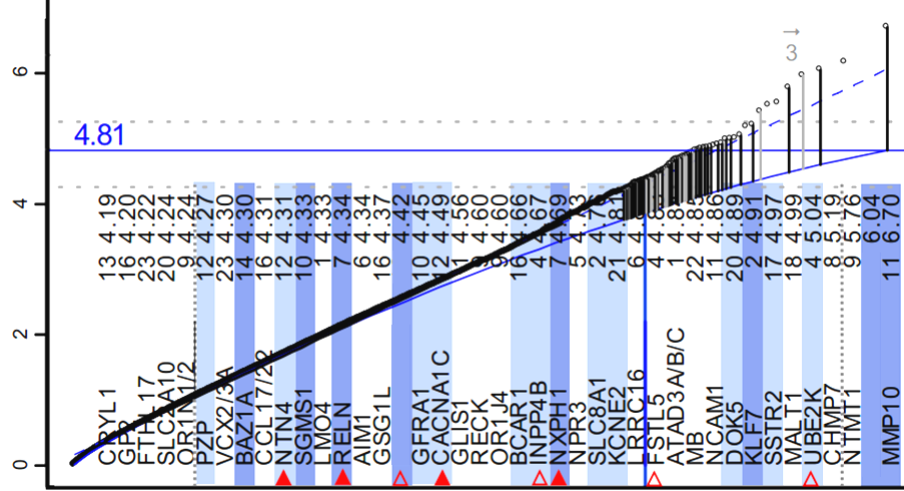
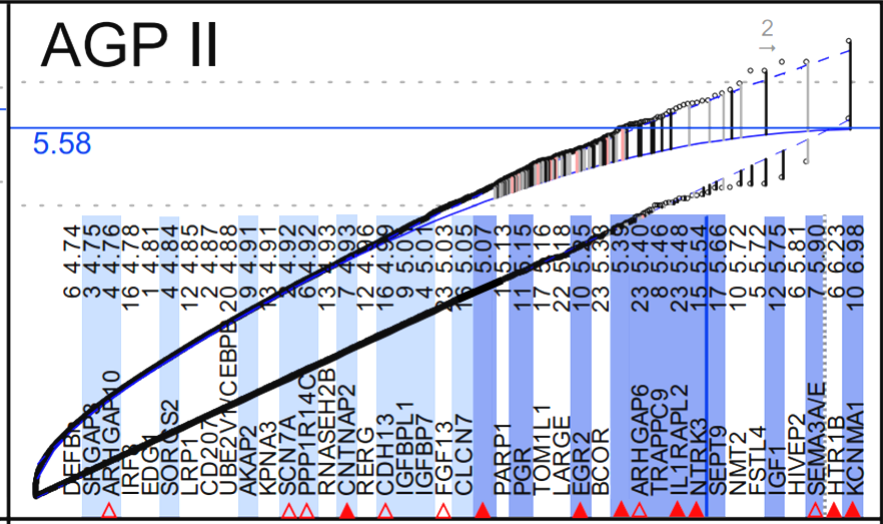
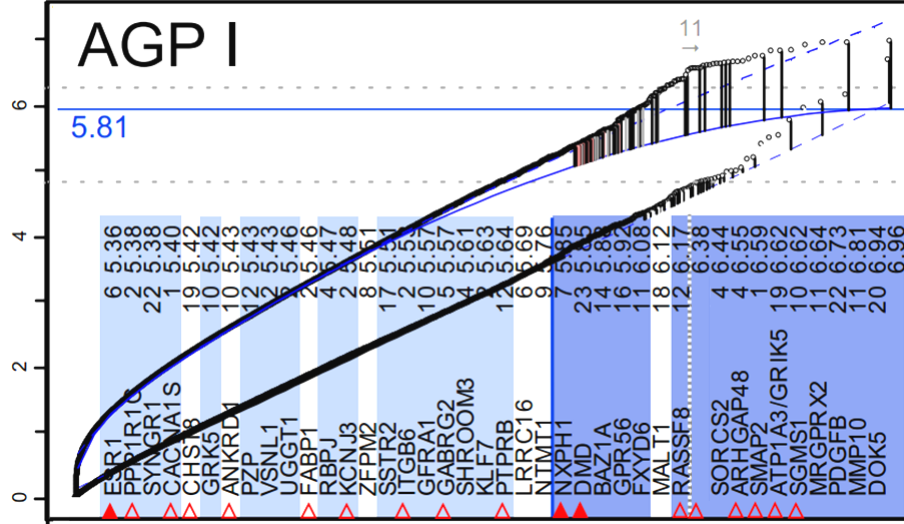


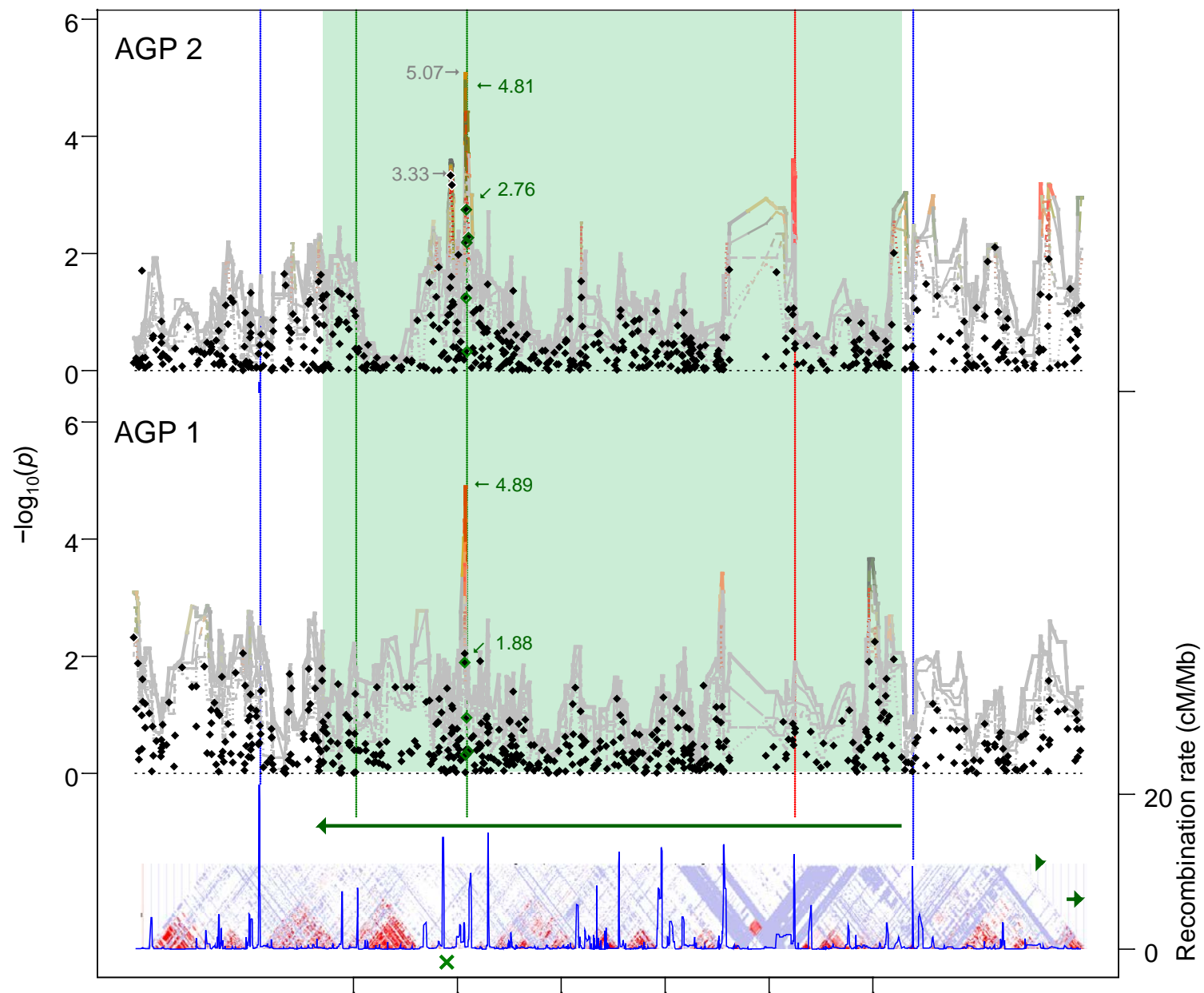
AGP I

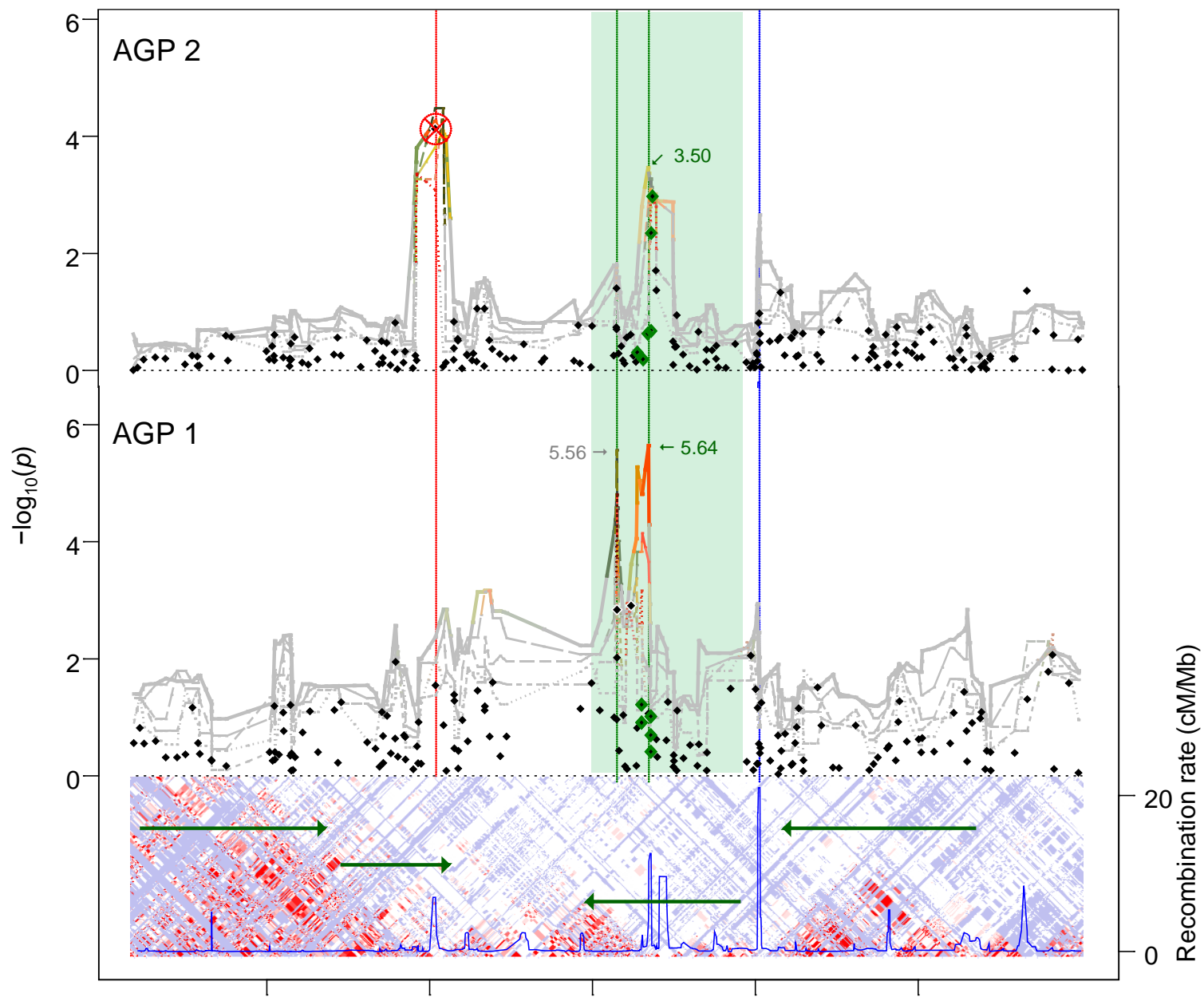


AGP II









Implications

Both phenotyping of AEs and GWAS have suffered from the use of less appropriate (univariate, linear model) statistical approaches.

As a novel computational biostat's approach, μ GWAS/ μ Phene can integrate information from

- complex phenotypes and
- wide loci (several neighboring SNPs)

from a few hundred subjects only.

Data collected during Phase III can now be used to

- Identify genetic risk factors for AEs and Tx failure,
- Suggest novel directions for drug development, and
- Guide with selection of study populations,

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The End

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